ECE 306L - FOURIER SERIES - LAB 2
THE SPECTRUMS OF PULSE TRAINS - PART I

FALL 2006 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to calculate and measure Fourier Series coefficients of periodic signals like pulse trains.

PRELAB

1. Sketch three cycles of a periodic signal $x(t)$ with period $T = 1\,\text{sec}$

2. What is the number of cycles/sec - the frequency - of a periodic signal with period $T = 1\,\text{msec}$

3. What is the frequency of a periodic signal that satisfies $x(t + 2 \times 10^{-4}) = x(t)$

4. Given $x(t) = 3\cos(2\pi 1000t)$
   a. What is the amplitude of $x(t)$
   b. What is the frequency of $x(t)$ in Hz
   c. What is the period of $x(t)$
   d. Use Matlab to obtain a graph of 3 cycles of $x(t)$
   e. Make use of your graph to verify that the period of $x(t)$ is $T = 1\,\text{msec}$
   f. Sketch the one-sided spectral plot of $x(t)$
   g. Sketch the two-sided spectral plot of $x(t)$

5. Given $x(t) = 3\cos(2\pi 1000t) + 2\cos(2\pi 2000t)$
   a. What is the frequency of $x(t)$ in Hz
   b. What is the period of $x(t)$
   c. Use Matlab to obtain a graph of $x(t)$
   d. Make use of your Matlab graph to verify your result for the period in part (b)
   e. Sketch the one-sided spectral plot of $x(t)$
   f. Sketch the two-sided spectral plot of $x(t)$

6. Given $x(t) = 1 + 3\cos(2\pi 1000t) + 2\cos(2\pi 2000t) + \cos(2\pi 3000t)$
   a. What is the frequency of $x(t)$ in Hz
   b. What is the period of $x(t)$
   c. Use Matlab to obtain a graph of $x(t)$
   d. Make use of your Matlab graph to verify your result for the period in part (b)
   e. Sketch the one-sided spectral plot of $x(t)$
   f. Sketch the two-sided spectral plot of $x(t)$

7. What is the frequency of the following periodic signal

$$x(t) = c_0 + \sum_{k=1}^{N} c_k \cos(2\pi k 1000\,t)$$

8. Generalizing on the result of Problem (7) we can show that if $x(t)$ is a sum of sinusoids with phases θ_k and kf_0 frequencies that are equal to integer multiples of f_o as follows

$$x(t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k)$$

then $x(t)$ is periodic with frequency f_o. If we then make use of Euler's Relation as follows

$$re^{j\theta} = r\cos\theta + jr\sin\theta \qquad r\cos\theta = \frac{r}{2}e^{j\theta} + \frac{r}{2}e^{-j\theta}$$

we can write our sum of sinusoids as a sum of complex exponentials as follows

$$x(t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_o t}$$

where

$$X_0 = c_0 \qquad X_{-k} = \frac{c_k}{2}e^{-j\theta_k} \qquad X_k = \frac{c_k}{2}e^{j\theta_k} \qquad k > 0$$

Express $x(t)$ as a sum of sinusoids if $f_o = 2\times 10^3$ Hz and $X_0 = 2$, $X_1 = 3e^{j1.2}$, $X_2 = 2e^{-j2}$

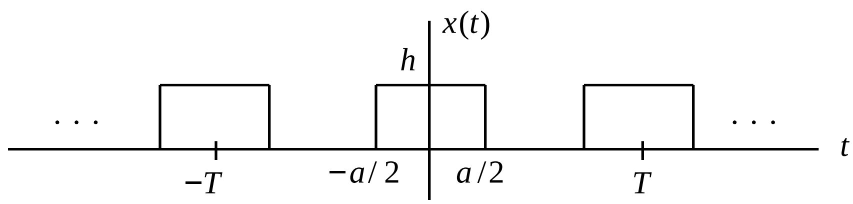
9. From Problem (8) we know that every signal $x(t)$ that is equal to a sum of sinusoids or equivalently a sum of complex exponentials as follows

$$x(t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_o t}$$

is periodic with period $T = 1/f_o$. What Fourier did was show the reverse. He showed that if $x(t)$ is periodic of period T then we can find a sum of sinusoids - or equivalently a sum of complex exponentials - that add up to $x(t)$ as follows

$$x(t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_o t}$$

In particular Fourier showed that if $x(t)$ is a periodic pulse train as follows



then $x(t)$ can be written as a sum of complex exponentials as follows

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_o t}$$

with $X_k = \frac{ha}{T}\text{sinc}(kf_o a)$ where $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$. Note that Matlab has the sinc function

a. Make use of Matlab to plot three cycles of the sum of the first five harmonics of our pulse train $x(t)$ as follows

$$x(t) \approx \sum_{k=-5}^{5} X_k e^{j2\pi k f_o t}$$

with $h = 2$, $a = 1\text{msec}$, $T = 2$ msec

b. How well does this finite sum approximate the pulse train
c. How would increasing the number of terms in the sum affect the approximation
d. Increase the number of terms to 10 and see what happens

10. The purpose of this problem is to draw the spectral plot of our pulse train. Given a 1/2 duty cycle pulse train $x(t)$ with $h = 2$, $a = 1$msec, $T = 2$msec
    a. Sketch the pulse train
    b. Make use of Matlab to obtain a graph of three lobes of the envelope $X_{env}(f)$ of the magnitudes of the Fourier Coefficients X_k of the two-sided spectral plot as follows

    $$|X_{env}(f)| = \left|\frac{ha}{T}\text{sinc}(fa)\right|$$

    Have Matlab draw the envelope with a dashed line
    c. Make use of the fact that the zero crossover frequencies - the frequencies where $\text{sinc}(fa) = 0$ - are at $f = \pm\frac{1}{a},\ \pm\frac{2}{a},\ \pm\frac{3}{a},\ \cdots$ to draw in by hand the magnitudes of the spectral lines $|X_k|$

11. Repeat Problem (10) for a 1/3 duty cycle pulse train with $h = 2$, $a = 2/3$ msec, $T = 2$msec

12. Repeat Problem (10) for a 1/4 duty cycle pulse train with $h = 2$, $a = 1/2$ msec, $T = 2$msec

13. Make use of Matlab to calculate the following Fourier coefficients

    $$c_k = 2\frac{ha}{T}|\text{sinc}(kf_o a)|$$

    in dBV for $k = 0$ to $k = 10$ for each of our pulse trains as follows
    a. 1/2 duty cycle pulse train in Problem 10
    b. 1/3 duty cycle pulse train in Problem 11
    c. 1/4 duty cycle pulse train in Problem 12

LAB

1. Display 5 to 10 cycles of a 1/2 duty cycle pulse train $x(t)$ with period $T_o = 1$ ms and peak-to-peak amplitude of 2 volts
    a. Measure the amplitude and frequency of $x(t)$. Save a screen capture for your report
    b. Display 5 to 10 cycles of the pulse train on the scope and then use the scope's FFT function to measure the frequencies and amplitudes (in dBV) of the first ten harmonics of its one-sided spectrum. Save screen captures for your report. Be sure to label your screen capture with the value of h, a and T_o

2. Repeat part (1) for a 1/3 duty cycle pulse train with period $T_o = 1$ ms and peak-to-peak amplitude of 2 volts

3. Repeat part (1) for a 1/4 duty cycle pulse train with period $T_o = 1$ ms and peak-to-peak amplitude of 2 volts

4. Repeat part (1) for a 1/5 duty cycle pulse train with period $T_o = 1$ ms and peak-to-peak amplitude of 2 volts

POSTLAB

1. Compare your calculated and measured spectrums of the pulse trains. Put your results in clearly labeled Tables. How well did things work

A Matlab program for calculating $x(t) = \sum_{k=-K}^{K} X_k e^{j2\pi k f_o t}$ of a given pulse train

```
h =2;
a=1e-3;
T=2e-3;
fo=1/T;
K=5;
k=-K:K;
Xk=(h*a/T)*sinc(k*a*fo);
N=1000;
t=linspace(0, 3*T, N);
x=zeros(1, N);
for n=1:N
    expn=exp(j*2*pi*k*fo*t(n);
    x(n)=sum(Xk.*expn);
end
plot (t, x)
```