ECE 209 - FIRST ORDER CIRCUITS - INVESTIGATION 9
BODE PLOTS OF FIRST ORDER TRANSFER FUNCTIONS

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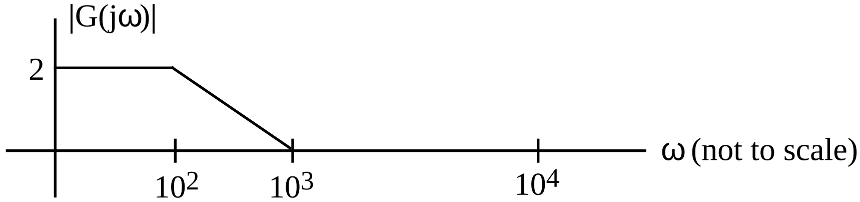
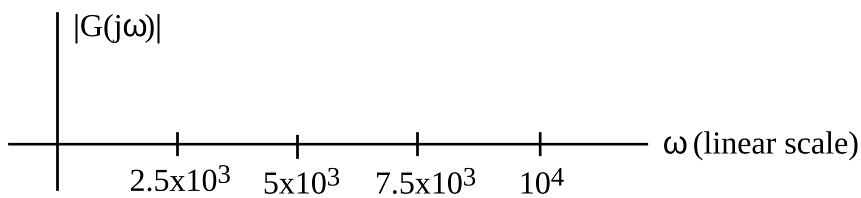
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to see the affects of scaling the axis of frequency response plots in different ways.

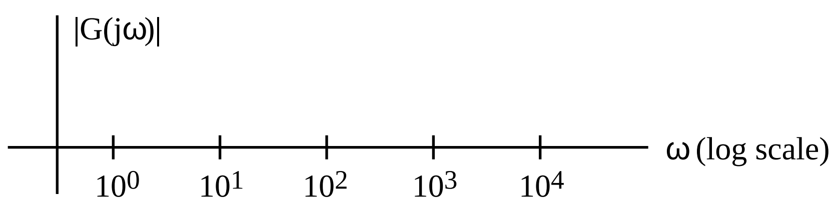
1. The objective of this problem is to demonstrate the affects of using a log scale for the frequency axis. Suppose we have a circuit with the following frequency response



   a. First sketch $|G(j\omega)|$ for a linearly scaled frequency axis as follows



   b. And then sketch $|G(j\omega)|$ with the frequency axis on a log scale as follows



   c. Describe the similarities and differences between the two scales. Discuss, in particular, the spacing of the frequencies and the "locations" of the origins
   d. On which graph is it easier to read the gains at $\omega = 100$ $\omega = 10^3$ and $\omega = 10^4$. Why
   e. What's the advantage of plotting ω on a log scale

2. The results of Problem (1) were for an idealized frequency response. Now let's take a look at some more realistic frequency responses
   a. Make use of Mathcad to obtain a graph of the magnitude of

$$G_1(j\omega) = \frac{100}{j\omega + 100}$$

over the frequency range $1 \le \omega \le 10^6$ first with the frequency axis on a linear scale and then on a log scale. Note that you can get your frequency axis to be plotted on a log scale by replacing every ω in $G(j\omega)$ by 10^{ω} and then plotting over the range $\omega = 0$ to $\omega = 6$

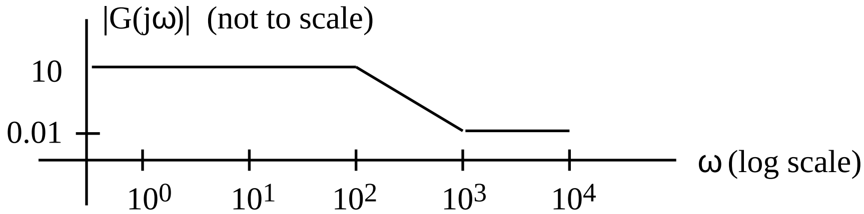
b. Make use of your graphs to estimate the frequency where the gain is equal to $(1/\sqrt{2}) = 0.707$ of its maximum value

c. Which graph was easier to read. Why

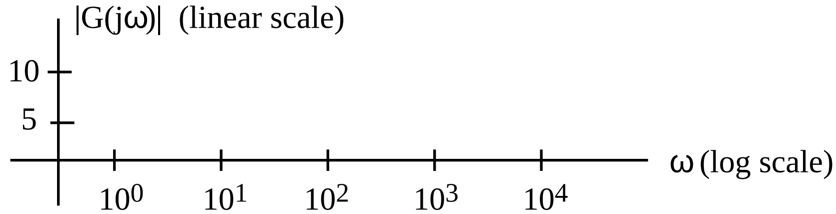
d. Repeat parts (a) - (c) for

$$G_2(j\omega) = \frac{j\omega}{j\omega + 10^4}$$

3. The objective of this problem is to demonstrate the affects of plotting $|G(j\omega)|$ on a log scale. Suppose we have a circuit with the following frequency response



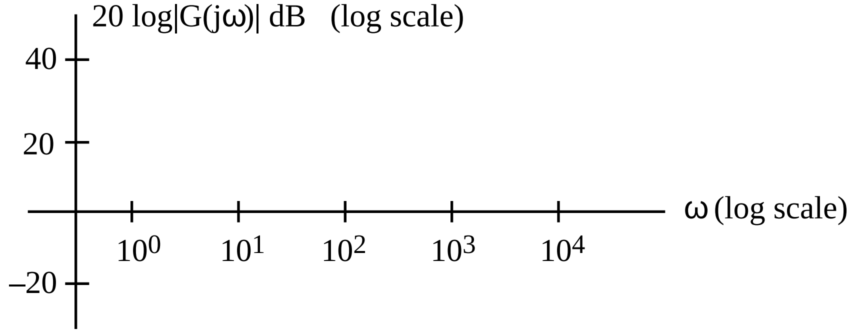
a. First sketch $|G(j\omega)|$ on a linear scale as follows



b. Before we sketch $|G(j\omega)|$ on a log scale we need to point out that by tradition we don't simply plot $\log|G(j\omega)|$. Instead we plot $20\log|G(j\omega)|$ in units of dB (decibels) after Alexander Graham Bell. Find the gains in dB when

(i) $|G| = 100$ (iii) $|G| = 1$
(ii) $|G| = 10$ (iv) $|G| = 0.1$

c. Now sketch $|G(j\omega)|$ for the graph at the beginning of this problem on a log scale in dB as follows



d. Describe the similarities and differences between the two scales

e. On which graph is it easier to determine the gain at $\omega = 1000$. Why

f. What's the advantage of plotting $|G(j\omega)|$ in dB. Note that we refer to plots of $20 \log |G(j\omega)|$ in dB versus $\log \omega$ as **Bode Plots**. You will see a lot of Bode Plots in electronics and control systems courses

4. The results of Problem (3) were for an idealized frequency response. Now let's take a look at more realistic frequency responses

   a. Make use of a plotting calculator or computer to obtain a Bode plot of the magnitude of

$$G_1(j\omega) = \frac{100}{j\omega + 100}$$

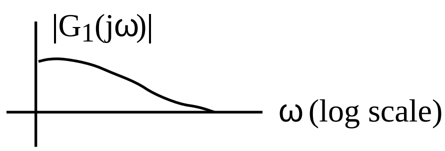
   over the frequency range $1 \le \omega \le 10^6$

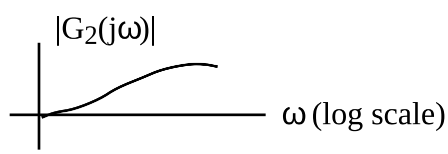
   b. Make use of your graph to estimate the frequency where the gain is equal to 0.01 of its maximum value. Explain how you got your result

   c. Repeat parts (a) and (b) for

$$G_2(j\omega) = \frac{j\omega}{j\omega + 10^4}$$

5. The gains of simple lowpass and highpass circuits like the ones we've been working with





have regions where the gains are relatively large and regions where they're relatively small. As a rule of thumb, we say that the boundaries between these regions are at the frequencies where the gains are $1/\sqrt{2}$ of their maximum values - the frequency where the gain is equal to

$$\frac{1}{\sqrt{2}} |G(j\omega)|_{\max}$$

where $|G(j\omega)|_{\max}$ is the maximum value of the gain. One of the main reasons we choose the factor $1/\sqrt{2}$ is that the corresponding frequencies turn out to be relatively straightforward to calculate - at least for simple circuits

   a. Do the calculations to show that

$$20 \log \left(\frac{1}{\sqrt{2}} |G(j\omega)|_{\max} \right) \approx 20 \log \left(|G(j\omega)|_{\max} \right) - 3\text{dB}$$

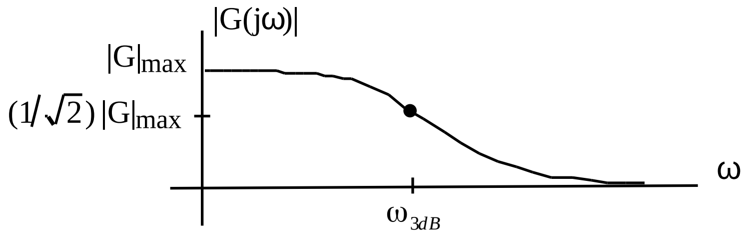
   We refer to a frequency where $|G(j\omega)|$ is $1/\sqrt{2}$ of its maximum value as a **3dB frequency** $\omega_{\mathbf{3dB}}$. Therefore

$$|G(j\omega_{3dB})| = \frac{1}{\sqrt{2}} |G(j\omega)|_{\max}$$

   and so

$$20 \log \left(|G(j\omega_{3dB})| \right) = 20 \log \left(|G(j\omega)|_{\max} \right) - 3 \text{ dB}$$

Memorize these last two relations. You will see ω_{3dB} in many applications. Graphically we have



b. Plot a Bode plot of the magnitude of the following transfer function

$$G(j\omega) = \frac{2000}{j\omega + 1000}$$

and then make use of it to find the 3dB frequency ω_{3dB}

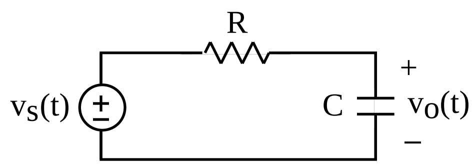
6. Show that the 3dB frequency ω_{3dB} of a first order lowpass with transfer function

$$G(j\omega) = \frac{K}{j\omega + a}$$

is $\omega_{3dB} = a$. **Memorize** this result. Verify that this is what you got in part (b) of Problem (5)

7. Make use of your result in Problem (6) to show that ω_{3dB} for a first order RC circuit is given by $\omega_{3dB} = 1/R_{TH}C$. **Memorize** this result

8. Given the following circuit



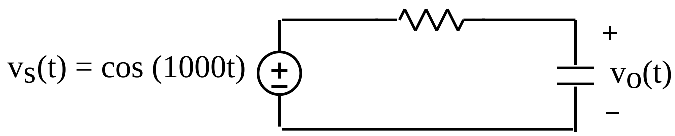
a. Find practical values for R and C so that $\omega_{3dB} = 10^4$
b. Find K of the transfer function
c. Sketch $|G(j\omega)|$ as a function of ω on a log scale

9. Show that the 3dB frequency ω_{3dB} of a first order highpass with transfer function

$$G(j\omega) = \frac{j\omega}{j\omega + a}$$

is $\omega_{3dB} = a$. **Memorize** this result

10. Given the following first order RC circuit



a. Find and sketch $v_O(t)$ when $\omega_{3dB} = 100$

b. Find and sketch $v_O(t)$ when $\omega_{3dB} = 1000$

c. Find and sketch $v_O(t)$ when $\omega_{3dB} = 10^4$

d. Describe what happens to the amplitude of $v_O(t)$ as the 3dB frequency of the low pass circuit increases. **Memorize** this relationship