

ECE 209 - FIRST ORDER CIRCUITS - INVESTIGATION 8 FREQUENCY RESPONSES OF FIRST ORDER RL CIRCUITS

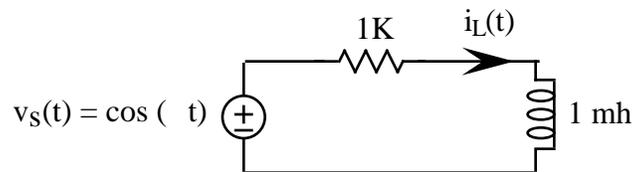
FALL 2000

A.P. FELZER

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to calculate the frequency responses of first order RL circuits just like we calculated them for first order RC circuits in the last investigation. We will also be defining what we mean by lowpass and highpass frequency responses.

1. Let us begin with the following first order RL circuit

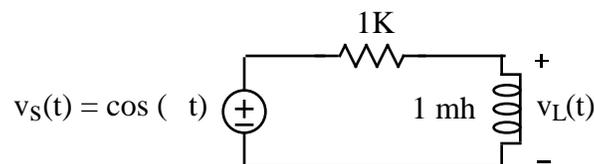


with sinusoidal steady state response $i_L(t)$ equal to

$$i_L(t) = \text{Re}[I_L(j\omega)e^{j\omega t}] = \text{Re}[|I_L(j\omega)|e^{j \angle I_L(j\omega)} e^{j\omega t}] = |I_L(j\omega)|\cos(\omega t + \angle I_L(j\omega))$$

- a. Find the steady state response of $i_L(t)$ when $\omega = 0$ and the input is constant.
- b. What will happen to the amplitude of the sinusoidal steady state $i_L(t)$ as ω increases.
Hint: How does increasing the frequency affect the amount of time the inductor's magnetic field has to build up.
- c. Find the steady state response of $i_L(t)$ when $\omega = 1$.
- d. Now make use of your results in parts (a)-(c) to sketch $|I_L(j\omega)|$ as a function of frequency ω . Describe your graph.
- e. Find and sketch $\angle I_L(j\omega)$ as a function of frequency ω .
- f. Reconcile any differences between your graphs in parts (d) and (e)

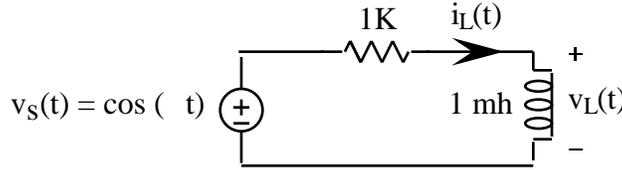
2. The objective of this problem is to continue with the circuit of Problem (1) as follows



- a. Find the steady state response of $v_L(t)$ when $\omega = 0$ and the input is constant.
- b. What will happen to the amplitude of the sinusoidal steady state $v_L(t)$ as ω increases.
How do you know.
- c. Find the steady state response of $v_L(t)$ when $\omega = 1$.

- d. Now make use of your results in parts (a)-(c) to sketch $|V_L(j\omega)|$ as a function of frequency ω . Describe your graph.
- e. Find and sketch $|V_L(j\omega)|$ as a function of frequency ω .
- f. Reconcile any differences between your graphs in parts (d) and (e)

3. In Problems (1) and (2) we found $I_L(j\omega)$ and $V_L(j\omega)$ in the following circuit

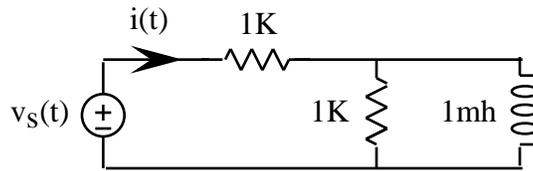


for the sinusoidal input $v_s(t) = \cos(\omega t)$. Now find the transfer functions $G_1(j\omega) = \frac{V_L(j\omega)}{V_s}$

and $G_2(j\omega) = \frac{I_L(j\omega)}{V_s}$ with respect to the phasor V_s for the more general input

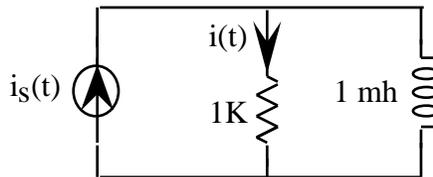
$$v_s(t) = A \cos(\omega t + \theta) = \text{Re}[V_s e^{j\omega t}]$$

4. The objective of this and the next problem is to calculate and make use of transfer functions of some other first order RL circuits. For the following circuit



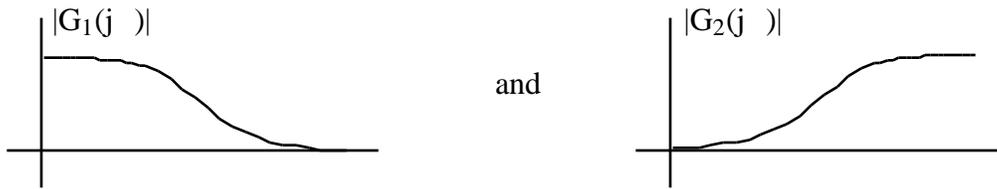
- a. Find the transfer function $G(j\omega) = \frac{I(j\omega)}{V_s}$
- b. Find and sketch $|G(j\omega)|$. Describe what's happening as ω increases.
- c. Find $i(t)$ when $v_s(t) = 5 \cos(1000t - 1.2)$

5. For the following circuit

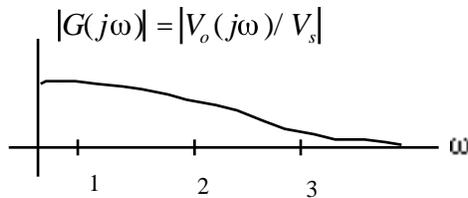


- a. Find the transfer function $G(j\omega) = \frac{I(j\omega)}{I_s}$
- b. Find and sketch $|G(j\omega)|$. Describe what's happening as ω increases.
- c. Find $i(t)$ when $i_s(t) = 5 \cos(1000t - 1.2)$ ma

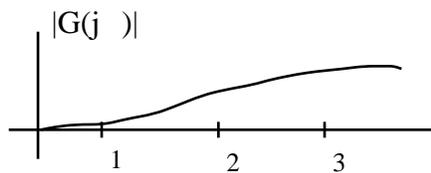
6. From the results in this and the last investigation we see that the magnitudes of the frequency responses of first order RC and RL circuits look like



- Why do we say $G_1(j\omega)$ is a **lowpass** response. **Memorize** this result.
 - And why do we say $G_2(j\omega)$ is a **highpass** response. **Memorize** this result.
7. Now suppose we have a circuit with a lowpass transfer function as follows



- Draw separate graphs of both $v_s(t) = \cos(\omega t)$ and the steady state $v_o(t)$ at the frequencies $\omega = 1, 2$ and 3 to illustrate what the steady state response of such a circuit looks like at low, medium and high frequencies.
 - Make use of your result in part (a) to explain why this circuit is referred to as a lowpass circuit.
- 8 And now suppose we have a circuit with a highpass transfer function as follows



- Draw separate graphs of both $v_s(t) = \cos(\omega t)$ and the steady state $v_o(t)$ at the frequencies $\omega = 1, 2$ and 3 to illustrate what the steady state response of such a circuit looks like at low, medium and high frequencies.
- Make use of your result in part (a) to explain why this circuit is referred to as a highpass circuit.