ECE 209 - FIRST ORDER CIRCUITS - INVESTIGATION 7
FREQUENCY RESPONSES OF FIRST ORDER RC CIRCUITS

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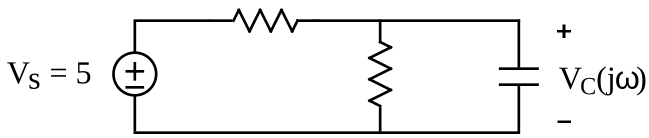
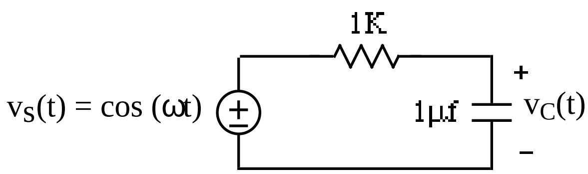
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We know from the last two investigations that phasor circuits like the following



have a lot in common with linear resistor circuits. But there is, of course, one fundamental difference - the sinusoidal steady state responses of circuits containing capacitors and inductors vary as a function of frequency while those containing only resistors do not. The objective of this investigation is to determine the **frequency responses** of first order RC circuits - to see how their sinusoidal steady state responses vary as functions of frequency.

1. Let us begin with the following simple first order RC circuit



with sinusoidal steady state response $v_C(t)$ equal to

$$v_C(t) = \mathrm{Re}\left[V_C(j\omega)e^{j\omega t}\right] = \mathrm{Re}\left[\left|V_C(j\omega)\right|e^{j\angle V_C(j\omega)}e^{j\omega t}\right] = \left|V_C(j\omega)\right|\cos\left(\omega t + \angle V_C(j\omega)\right)$$

a. Find the steady state response of $v_C(t)$ when $\omega = 0$ and the input is constant.

b. What will happen to the amplitude of the sinusoidal steady state $v_C(t)$ as ω increases.

Hint: How does increasing the frequency ω affect the amount of time the equivalent positive charges have to accumulate on the plates of the capacitor. And how, in turn, does this affect the amount of charge that can accumulate during any given cycle of the input $v_S(t)$.

c. Find the steady state response of $v_C(t)$ when $\omega = \infty$

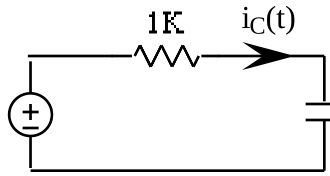
d. Make use of your results in parts (a)-(c) to sketch $|V_C(j\omega)|$ as a function of frequency ω. Describe your graph.

e. Now find $V_C(j\omega)$

f. Then make use of your result in part (e) to sketch $|V_C(j\omega)|$ as a function of frequency ω

g. Reconcile any differences between your graphs in parts (d) and (f)

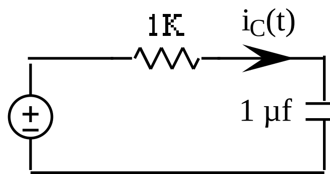
2. The objective of this problem is to continue with the circuit of Problem (1) as follows



to find how the sinusoidal steady state response of $i_C(t)$ varies as a function of frequency.

a. Find the steady state response of $i_C(t)$ when $\omega = 0$ and the input is constant.
b. What will happen to the amplitude of the sinusoidal steady state $i_C(t)$ as ω increases. How do you know.
c. Find the steady state response of $i_C(t)$ when $\omega = \infty$
d. Make use of your results in parts (a)-(c) to sketch $|I_C(j\omega)|$ as a function of frequency ω. Describe your graph.
e. Now find $V_C(j\omega)$
f. Then make use of your result in part (e) to sketch $|I_C(j\omega)|$ as a function of frequency ω
g. Reconcile any differences between your graphs in parts (d) and (f)

3. In Problems (1) and (2) we found $V_C(j\omega)$ and $I_C(j\omega)$ in the following circuit



for the sinusoidal input $v_S(t) = \cos(\omega t)$. Now find $V_C(j\omega)$ and $I_C(j\omega)$ in terms of the phasor V_S for the same circuit but with the more general input

$$v_s(t) = A\cos(\omega t + \theta) = \mathrm{Re}\left[V_s e^{j\omega t}\right]$$

4. Generalizing on the result of Problem (3) it can be shown that if a linear circuit is in the sinusoidal steady state then each of the voltage and current phasors will be proportional to the phasor of the input. For the circuit in Problems (1) and (2), in particular, we have

$$V_C(j\omega) = G_1(j\omega)V_s \quad \text{and} \quad I_C(j\omega) = G_2(j\omega)V_s$$

Note that we refer to the $G(j\omega)$'s as **transfer functions** of the circuit - and that we write them as ratios of polynomials in jw as follows

$$G(j\omega) = \frac{j\omega}{j\omega + 1000}$$

a. What's the transfer function $G(j\omega) = V_o(j\omega)/V_s$ of a circuit with

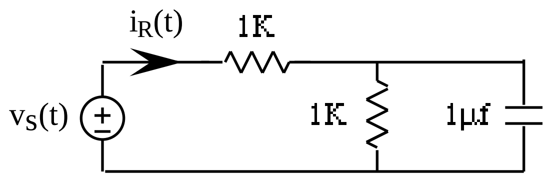
$$V_o(j\omega) = \frac{j\omega}{j\omega + 1000} V_s$$

b. What's the transfer function $G(j\omega) = I_o(j\omega)/I_s$ of a circuit with

$$I_o(j\omega) = \frac{1000}{j\omega + 1000} I_s$$

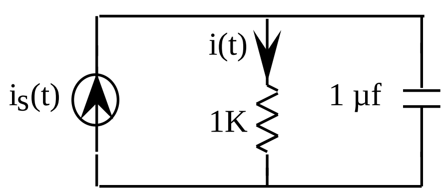
c. What's the sinusoidal steady state response of the circuit in part (a) with input $v_S(t) = 5 \cos(1000t + 1)$
d. What's the sinusoidal steady state response of the circuit in part (b) with input $i_S(t) = 3 \cos(1000t - 1)$ ma
e. How are transfer functions helpful - why do we go to the trouble of calculating them

5. For the following circuit



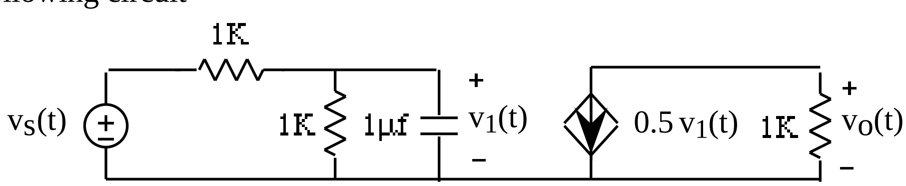
a. Find the transfer function $G(j\omega) = \frac{I_R(j\omega)}{V_s}$
b. Find and sketch $|G(j\omega)|$. Describe what's happening as ω increases.
c. Find $i_R(t)$ when $v_S(t) = 5 \cos(1000t - 1.2)$

6. For the following circuit



a. Find the transfer function $G(j\omega) = \frac{I(j\omega)}{I_s}$
b. Find and sketch $|G(j\omega)|$. Describe what's happening as ω increases.
c. Find i(t) when $i_S(t) = 5 \cos(1000t - 1.2)$ ma

7. For the following circuit



a. Find the transfer function $G(j\omega) = \frac{V_o(j\omega)}{V_s}$
b. Find and sketch $|G(j\omega)|$. Describe what's happening as ω increases.
c. Find $v_O(t)$ when $v_S(t) = 5 \cos(1000t - 1.2)$