ECE 209 - PHASOR CIRCUITS - INVESTIGATION 6
BASIC PROPERTIES OF PHASOR CIRCUITS

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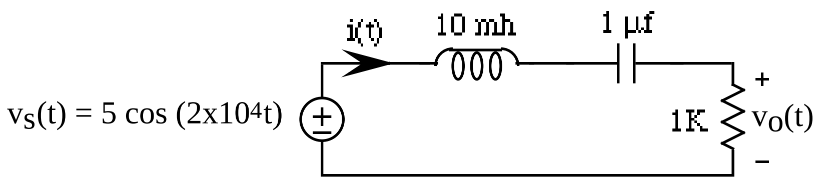
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

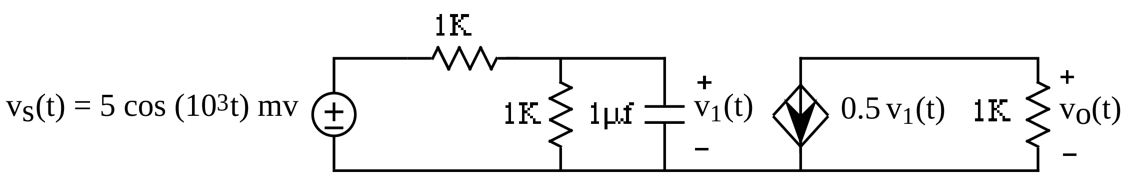
From the last investigation we know that even though phasor circuits are not real - we can't build them in the lab - they're *mathematically equivalent* to real circuits. Which is really nice since they're so much easier to analyze for sinusoidal steady state responses than real circuits. In particular the analysis of phasor circuits is very much like the analysis of linear resistor circuits except, of course, for the presence of complex numbers. The objective of this investigation is to demonstrate that phasor circuits also have many of the same properties that linear resistor circuits do. We begin with some review problems.

1. The objective of this and the next problem is to practice analyzing general phasor circuits. Given the following circuit



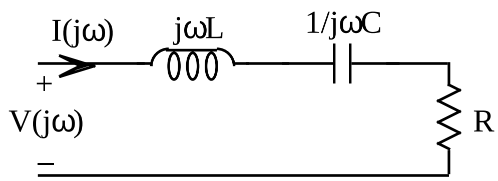
   a. Draw the phasor circuit
   b. Find $I(j2x10^4)$
   c. Make use of your result in part (b) to find $V_O(j2x10^4)$
   d. Find the sinusoidal steady state $v_O(t)$

2. Given the following circuit



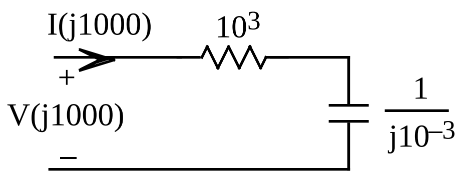
   a. Draw the phasor circuit
   b. Write the node equations and then put them in matrix form
   c. Then find $V_O(j10^3)$ and the corresponding steady state $v_O(t)$

3. The objective of this problem is to show that the equivalent impedances $Z(j\omega)$ of R's, L's and C's connected in series as follows



is simply the sum of the individual impedances

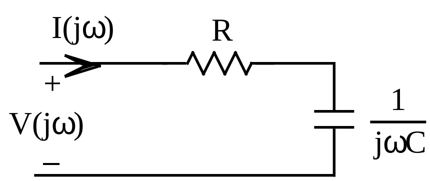
a. To find the equivalent impedance of a phasor circuit like that above we need to find V(jω) as a function of I(jω) and then solve for Z(jω) = V(jω)/I(jω). Make use of KVL to find V(j1000) as a function of I(j1000) in the following phasor circuit



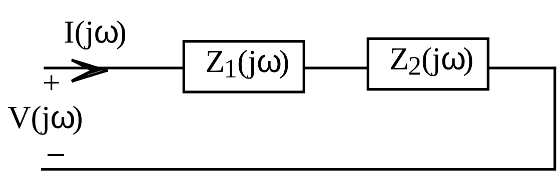
and then solve for the equivalent impedance

$$Z(j1000) = \frac{V(j1000)}{I(j1000)}$$

b. Now make use of KVL to find the equivalent impedance Z(jω) = V(jω)/I(jω) of the more general phasor circuit



c. And finally show that the equivalent impedance of any two impedances in series as follows

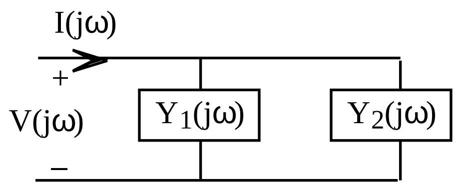


is simply the sum of the impedances. Then verify that the equivalent impedances you calculated in parts (a) and (b) are in fact the sums of the impedances of the resistor and capacitor.

d. What is the equivalent impedance of n impedances $Z_1(j\omega)$, $Z_2(j\omega)$, . . . , $Z_n(j\omega)$ connected in series

e. How are your results in this problem similar to the results for series resistor circuits

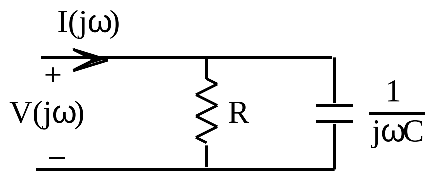
4. Now let's suppose that we have two general admittances in parallel as follows



a. What would you expect is the equivalent admittance $Y(j\omega) = I(j\omega)/V(j\omega) = 1/Z(j\omega)$ of $Y_1(j\omega)$ and $Y_2(j\omega)$ connected in parallel in this circuit

b. Make use of node equations to find the equivalent admittance $Y(j\omega)$ of the circuit. Are your results what you expected in part (a). If not, explain what in fact is going on. How is your result similar to the expression for the equivalent conductance of resistors in parallel

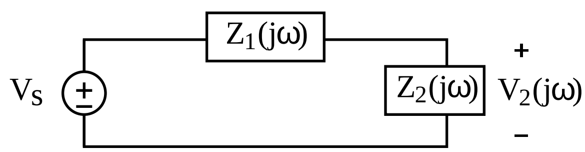
c. Find the equivalent admittance $Y(j\omega)$ of the following phasor circuit



d. Make use of your result in part (b) to find the equivalent impedance $Z(j\omega) = 1/Y(j\omega)$ of our general circuit above in terms in of $Z_1(j\omega)$ and $Z_2(j\omega)$

e. What is the equivalent admittance $Y(j\omega)$ of n admittances $Y_1(j\omega)$, $Y_2(j\omega)$, . . . , $Y_n(j\omega)$ connected in parallel.

5. The objective of this problem is to derive the voltage division equation for series phasor circuits. Make use of node equations to show that $V_2(j\omega)$ in the following series phasor circuit

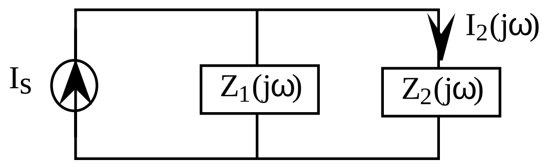


is given by

$$V_2(j\omega) = \frac{Z_2(j\omega)}{Z_1(j\omega) + Z_2(j\omega)} V_s$$

Memorize this result. How is voltage division for phasor circuits similar to that for resistor circuits.

6. The objective of this problem is to derive the current division equation for parallel phasor circuits. Make use of node equations to show that $I_2(j\omega)$ in the following phasor circuit

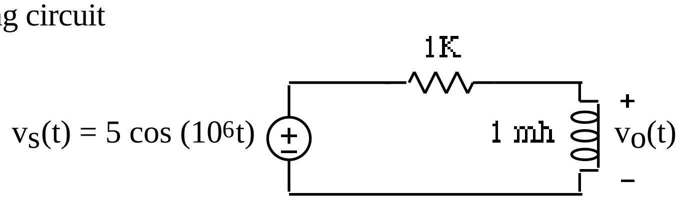


is given by

$$I_2(j\omega) = \frac{Z_1(j\omega)}{Z_1(j\omega) + Z_2(j\omega)} I_s = \frac{Y_2(j\omega)}{Y_1(j\omega) + Y_2(j\omega)} I_s$$

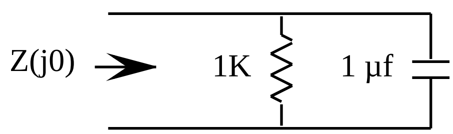
Memorize this result. How is current division for phasor circuits similar to that for resistor circuits.

7. Given the following circuit



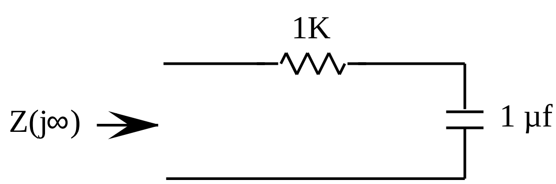
a. Draw the phasor circuit
b. Verify that both node equations and voltage division give the same $V_O(j10^6)$

8. Find the equivalent impedance of the following circuit at DC when $\omega = 0$.



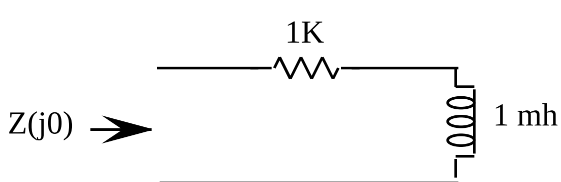
Hint - make use of the fact from Investigation 4 that a circuit element with infinite impedance is equivalent to an open circuit. Be sure to draw the equivalent circuit.

9. Find the equivalent impedance of the following circuit when $\omega = \infty$.



Hint - make use of the fact from Investigation 4 that a circuit element with zero impedance is equivalent to a short circuit. Be sure to draw the equivalent circuit.

10. Find the equivalent impedance of the following circuit at DC when $\omega = 0$.



11. Find the equivalent impedance of the following circuit when $\omega = \infty$.