ECE 209 - PHASOR CIRCUITS - INVESTIGATION 4
IMPEDANCES, ADMITTANCES AND KIRCHHOFF'S LAWS

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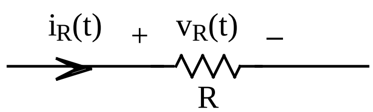
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

As we saw in the last investigation, every sinusoidal voltage and every sinusoidal current has associated with it a corresponding phasor $V(j\omega)$ and $I(j\omega)$ respectively. The goals of this investigation are to find out how the voltage and current phasors of resistors, capacitors and inductors are related to each other. And then to explore Kirchhoff's Laws for phasors. The plan is to start with time domain relations like we did in the previous investigations and then use Euler's Relation to express the sinusoids in terms of complex exponentials.

1. The objective of this first problem is to determine how $V(j\omega)$ is related to $I(j\omega)$ for resistors like the following



when $v_R(t)$ and $i_R(t)$ are sinusoids of frequency ω. We begin with a specific example and then generalize.

a. First find $v_R(t) = Ri_R(t)$ for a 2K resistor with sinusoidal current

$$i_R(t) = 5\cos\left(10^3t - \pi/6\right) \text{ ma} = \text{Re}\left[5x10^{-3}\, e^{-j\pi/6}\, e^{j1000t}\right]$$

and then make use of the result to find the corresponding voltage phasor $V_R(j1000)$

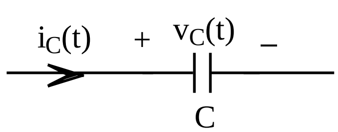
b. Next find $v_R(t) = Ri_R(t)$ for a general resistor of value R with sinusoidal current

$$i_R(t) = A\cos\left(10^3t + \theta\right) = \text{Re}\left[\, I_R(j1000)e^{j1000t}\,\right]$$

and then make use of the result to find the corresponding voltage phasor $V_R(j1000)$ as a function of $I_R(j1000)$.

c. And finally make use of $v_R(t) = Ri_R(t)$ to find $V_R(j\omega)$ as a function of $I_R(j\omega)$ for a resistor of value R with sinusoidal current $i_R(t) = A\cos(\omega t + \theta) = \text{Re}\left[\, I_R(j\omega)e^{j\omega t}\,\right]$ and sinusoidal voltage $v_R(t) = B\cos(\omega t + \theta) = \text{Re}\left[\, V_R(j\omega)e^{j\omega t}\,\right]$. **Memorize** this result.

2. The objective of this problem is to determine how $V(j\omega)$ is related to $I(j\omega)$ for capacitors like the following



when $v_C(t)$ and $i_C(t)$ are sinusoids of frequency ω. As we did for resistors, we begin with a specific example and then generalize.

a. First find $i_C(t) = C\dfrac{dv_C(t)}{dt}$ for a 1 μf capacitor with sinusoidal voltage

$$v_C(t) = 5\cos\left(10^3 t + \pi/6\right) = \text{Re}\left[5e^{j\pi/6}\, e^{j1000t}\right] \text{ volts}$$

and then make use of the result to find the corresponding current phasor $I_C(j1000)$.

b. Next find $i_C(t) = C\dfrac{dv_C(t)}{dt}$ for a general capacitor of value C with sinusoidal voltage

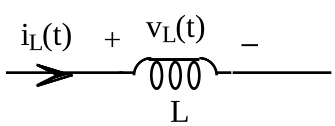
$$v_C(t) = A\cos\left(10^3 t + \theta\right) = \text{Re}\left[V_C(j1000)e^{j1000t}\right] \text{ volts}$$

and then make use of the result to find the corresponding current phasor $I_C(j1000)$ as a function of $V_C(j1000)$.

c. And finally make use of $i_C(t) = C\dfrac{dv_C(t)}{dt}$ to find $I_C(j\omega)$ as a function of $V_C(j\omega)$ for a capacitor of value C with sinusoidal voltage $v_C(t) = A\cos(\omega t + \theta) = \text{Re}\left[V_C(j\omega)e^{j\omega t}\right]$ and sinusoidal current $i_C(t) = B\cos(\omega t + \varphi) = \text{Re}\left[I_C(j\omega)e^{j\omega t}\right]$. **Memorize** this result.

d. Sketch $|V_C(j\omega)|$ as a function of frequency ω for a 1μf capacitor with current $i_C(t) = 5\cos(\omega t)$ ma. Describe what's going on.

3. The objective of this problem is to determine how $V(j\omega)$ is related to $I(j\omega)$ for capacitors like the following



$$v_L(t) = L\frac{di_L(t)}{dt}$$

when $v_L(t)$ and $i_L(t)$ are sinusoids of frequency ω.

a. Make use of the time domain relation for inductors $v_L(t) = L\dfrac{di_L(t)}{dt}$ to find $V_L(j\omega)$ as a function of $I_L(j\omega)$. **Memorize** this result.

b. Sketch $|I(j\omega)|$ as a function of frequency ω for a 1mh inductor with voltage $v(t) = 5\cos\omega t$. Describe what's going on.

4. Make use of your results in Problems (1)–(3) to find the **impedances** $Z(j\omega)$ of resistors, capacitors and inductors as given by

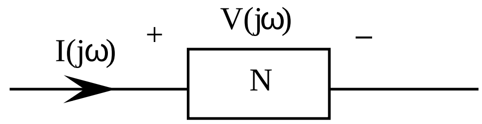
$$Z(j\omega) \equiv \frac{V(j\omega)}{I(j\omega)}$$

and the **admittances** $Y(j\omega)$ as given by

$$Y(j\omega) \equiv \frac{1}{Z(j\omega)} = \frac{I(j\omega)}{V(j\omega)}$$

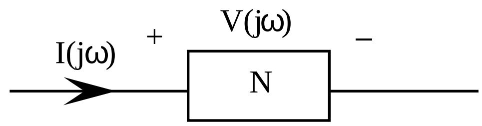
Put your results in a Table. **Memorize** them.

5. The objective of this and the next problem is to look at two special impedances. First justify that if the impedance of a circuit element N as follows



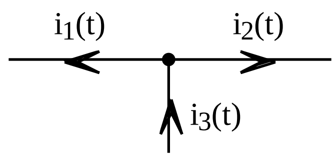
is zero then N is equivalent to a short circuit. **Memorize** this result forever.

6. Now justify that if the impedance of a circuit element N as follows



is infinite then N is equivalent to an open circuit. **Memorize** this result forever.

7. The objective of this problem is to demonstrate Kirchhoff's Current Law for phasor circuits. Let us begin with the following node N



with

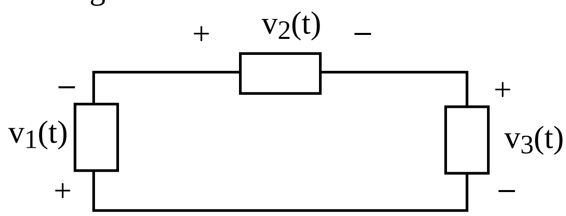
$$i_1(t) = 5 \cos\left(10^3 t - 2.6\right) \text{ ma}$$

$$i_2(t) = 2 \cos\left(10^3 t + 1\right) \text{ ma}$$

$$i_3(t) = 3.33 \cos\left(10^3 t - 2.87\right) \text{ ma}$$

a. By KCL these currents should satisfy $i_1(t) + i_2(t) - i_3(t) = 0$ at each time *t* . Verify that they do in fact satisfy KCL (at least to within roundoff error) by plotting a graph of $i_1(t) + i_2(t) - i_3(t)$.
b. Would you expect the magnitudes (amplitudes) of these currents to satisfy KCL. See if they do. Explain what's going on.
c. Would you expect the phasors of these currents to satisfy KCL. See what happens. Reconcile your results with your expectations.
d. What would you expect, based on this example, is the relationship between current phasors at arbitrary nodes.

8. The objective of this problem is to demonstrate Kirchhoff's Voltage Law for phasor circuits. Let us begin with the following circuit



with

$v_1(t) = 3 \cos\left(10^3 t + 1\right)$ volts

$v_2(t) = 2 \cos\left(10^3 t - 0.5\right)$ volts

$v_3(t) = 3.7 \cos\left(10^3 t - 2.71\right)$ volts

a. By KVL these voltages should satisfy $v_1(t) + v_2(t) + v_3(t) = 0$ at each time *t* . Verify that they do in fact satisfy KVL (at least to within roundoff error) by plotting a graph of $v_1(t) + v_2(t) + v_3(t)$.
b. Would you expect the magnitudes (amplitudes) of these voltages to satisfy KVL. See if they do. Explain what's going on.
c. Would you expect the phasors of these voltages to satisfy KVL. See what happens. Reconcile your results with your expectations.
d. What would you expect, based on this example, is the relationship between voltage phasors around arbitrary closed loops.