ECE 209 - FOURIER SERIES - INVESTIGATION 24
STEADY STATE RESPONSES TO PERIODIC INPUTS - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

As we've seen in the last several investigations on Fourier Series, periodic signals x(t) of frequency f_o can be expressed as sums of sinusoids at integer multiples of the fundamental frequency f_o as follows

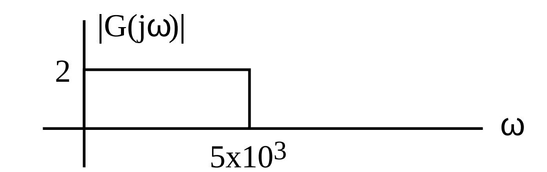
$$x(t) = c_o + c_1 \cos(2\pi f_o t + \theta_1) + c_2 \cos(2\pi 2 f_o t + \theta_2) + \ . \ . \ .$$

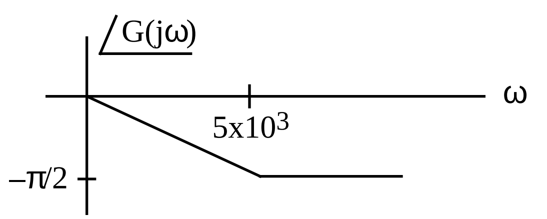
This is particularly nice because it means we can use superposition to find the steady state responses of linear circuits to periodic signals x(t) as follows:

(a) First calculate the Fourier Series expansion of x(t)
(b) Then calculate the sinusoidal steady state response to each of the harmonics
(c) And then add together the sinusoidal steady state responses to the harmonics

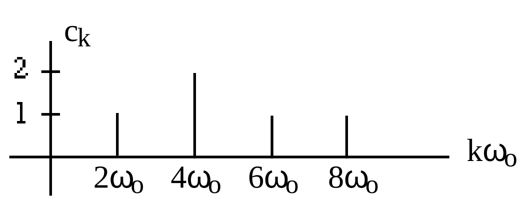
We call this scheme **frequency domain analysis**. **Memorize** it. The objective of this investigation is to get some practice with frequency domain analysis.

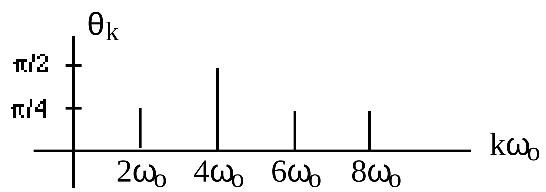
1. Suppose a circuit with the following lowpass frequency response





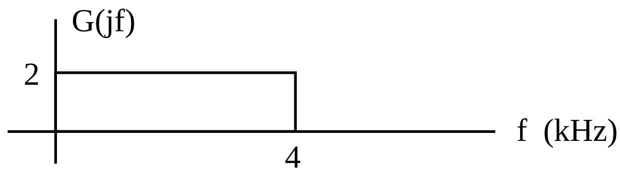
has a periodic input x(t) of fundamental frequency $\omega_o = 10^3$ and spectral plot as follows



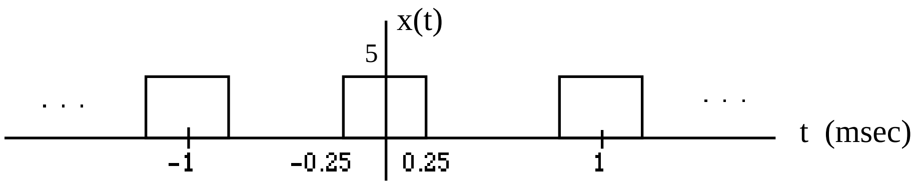


a. Which harmonics are in the passband and which in the stopband of the filter
b. Sketch the spectral plot of the output y(t)
c. Find y(t) as a sum of sinusoids

2. Suppose a circuit with the following lowpass frequency response



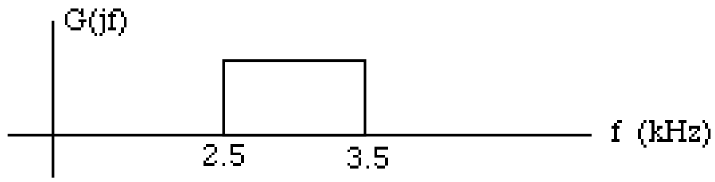
has a pulse train input as follows



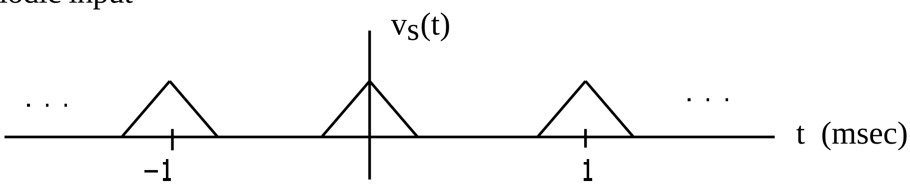
a. Make a spectral plot of the input pulse train x(t)
a. Make a spectral plot of the steady state response y(t)
b. Express y(t) as a sum of sinusoids
c. Make use of Mathcad to obtain a graph of the steady state response y(t)

3. Draw the ideal frequency response of a circuit that will only pass the DC term and first five harmonics of a periodic signal of period $T = 1$ msec

4. Sketch the steady state response of a circuit with the following frequency response



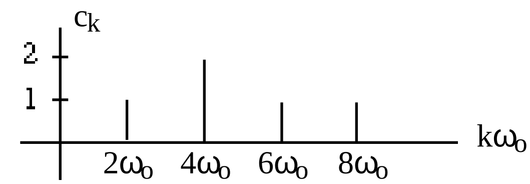
to the periodic input

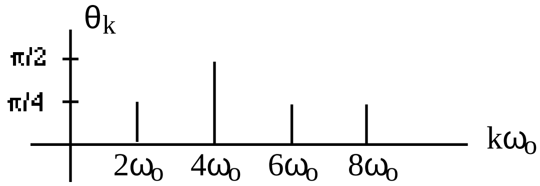


5. Suppose a circuit with transfer function

$$G(j\omega) = \frac{10^4 (j\omega)}{(j\omega)^2 + 5\times 10^3 (j\omega) + 2.5\times 10^7}$$

has a periodic input of fundamental frequency $\omega_o = 10^3$ with the following spectral plot





a. Sketch the spectral plot of the steady state output y(t)
b. Express y(t) as a sum of sinusoids

6. Generalizing on the results of the problems in this Investigation we see that if

$$d_k \cos(k\omega_o t + \phi_o)$$

is the steady state response to the k'th harmonic $c_k \cos(k\omega_o t + \theta_o)$ of a periodic input x(t) then the corresponding phasors are related by

$$Y(jk\omega_o) = G(jk\omega_o)\, X(jk\omega_o)$$

and so

$$d_0 = Y(j0) = c_0 G(j0)$$
$$d_k = |Y(jk\omega_o)| = |G(jk\omega_o)X(jk\omega_o)| = |G(jk\omega_o)||X(jk\omega_o)| = c_k|G(jk\omega_o)|$$
$$\phi_k = \quad Y(jk\omega_o) = \quad G(jk\omega_o)X(jk\omega_o) = \quad G(jk\omega_o) + \quad X(jk\omega_o) = \quad G(jk\omega_o) + \theta_k$$

Make use of these relationships to express the steady state response y(t)

$$y(t) = d_o + \sum_{k=1}^{\infty} d_k \cos(k\omega_o t + \phi_k)$$

as a function of c_k, θ_k and $G(jk\omega_o)$