ECE 209 - FOURIER SERIES - INVESTIGATION 22
THE SPECTRUMS OF PERIODIC SIGNALS - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last investigation we know that if x(t) is periodic with fundamental frequency f_o then we can express x(t) as a sum of sinusoids as follows

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k)$$

with

$$c_k = \sqrt{a_k^2 + b_k^2} \qquad \text{and} \qquad \theta_k = \tan^{-1}\left(\frac{-b_k}{a_k}\right)$$

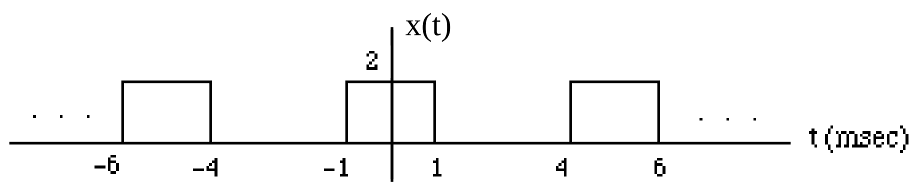
where

$$a_0 = c_o = \frac{1}{T}\int_T x(t)dt$$

$$a_k = \frac{2}{T}\int_T x(t)\cos(2\pi k f_o t)dt \qquad b_k = \frac{2}{T}\int_T x(t)\sin(2\pi k f_o t)dt$$

The main objective of this investigation is to practice calculating Fourier Series expansions and sketching corresponding spectral plots.

1. Given the same pulse train as in the previous investigation

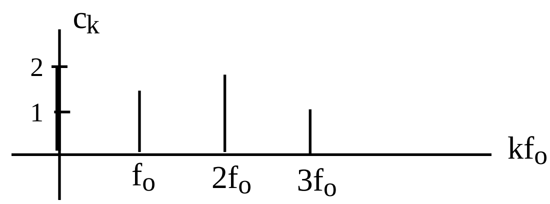


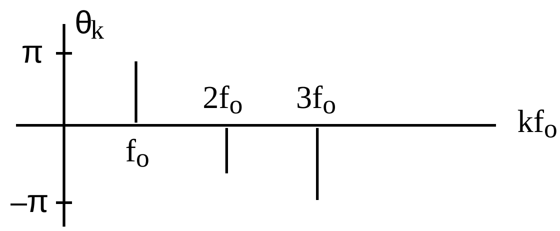
   a. Evaluate the integrals you came up with in the previous investigation to obtain values for the Fourier Coefficients $a_o, a_1, b_1, a_2, b_2, \cdots, a_5, b_5$
   b. Then calculate the values for $c_o, c_1, \theta_1, \ldots, c_5, \theta_5$
   c. The **spectrum** of a periodic signal like x(t) is by definition simply the amplitudes and phases of its harmonics. To "see" the spectrum of a periodic signal we simply graph its spectral plot. Sketch the spectrum of x(t) through the first five harmonics.
   d. Use Mathcad to obtain a graph of

$$x(t) \approx c_o + \sum_{k=1}^{5} c_k \cos(k\omega_o t + \theta_k)$$

   Your graph should "strongly resemble" the pulse train. Optional - see what happens as more terms are added.

2. Find x(t) for a periodic signal of frequency $f_o = 2\,\text{KHz}$ with the following spectral plot





3. Draw an arbitrary spectral plot of a periodic signal of fundamental frequency $f_o = 1\,\text{KHz}$. Then make use of Mathcad to obtain a graph of what you've created.

4. From the Investigations on Average Power we know that the average power of a resistor R with the following periodic voltage

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos(k\omega_o t + \theta_k) \qquad \text{is given by} \qquad P_{av} = \frac{c_o^2}{R} + \sum_{k=1}^{\infty} \frac{c_k^2}{2R} = \sum_{k=0}^{\infty} P_k^2$$

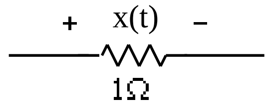
The corresponding average **normalized power** is by definition the average power when $R = 1\Omega$

   a. Find the equation for average normalized power
   b. Draw a **power spectral plot** - a plot of the normalized powers P_k of the harmonics as a function of nf_o when x(t) is equal to

$$x(t) = 3 + 2\cos\big((2\pi f_o)t + 1.3\big) + 1.5\cos\big(2(2\pi f_o)t - 0.5\big) + 2\cos\big(3(2\pi f_o)t\big)$$

   c. What is the total normalized average power of x(t) in part (b)

5. Suppose the pulse train of Problem (1) is the voltage v(t) across a 1Ω resistor as follows



   a. First sketch $x^2(t)$ and then make use of $P_{av} = \frac{1}{T}\int_T x^2(t)\,dt$ to find the resistor's average power P_{av}
   b. Then find the average powers P_k of the constant term c_o and the the first five harmonics.
   c. How close does the sum of the powers of the constant term and the first five harmonics come to the actual value of P_{av} you calculated in part (a).