

# ECE 209 - FOURIER SERIES - INVESTIGATION 20

## CHARACTERISTICS OF PERIODIC SIGNALS

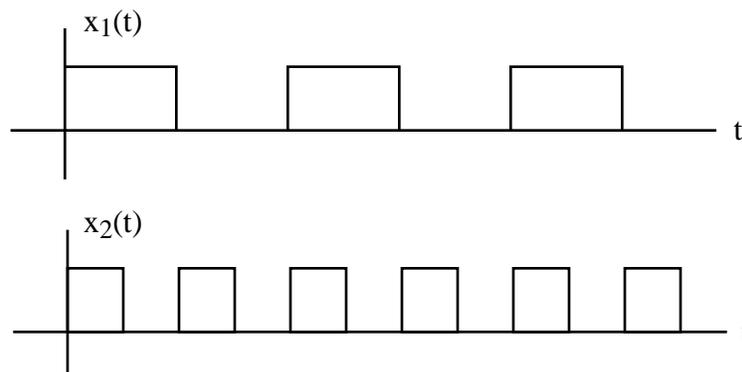
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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

At this point we know how to make use of phasor circuits and transfer functions to algebraically find the steady state responses of linear circuits to sinusoids and sums of sinusoids. The objective of this and the next five investigations on Fourier Series is to show how these algebraic techniques can be used to find the steady state responses of linear circuits to general periodic signals like pulse trains. The objective of this first investigation on Fourier Series is to investigate the basic properties of periodic signals.

1. What do we mean when we say a signal is periodic. Draw some
2. What do we mean by the period  $T$  of a periodic signal
3. Draw a periodic signal with period  $T = 1$  msec
4. What do we mean by the frequency in cycles/sec or Hertz (**Hz**) of a periodic signal. Draw a periodic signal of low frequency and then one of high frequency. Describe the difference.
5. What is the frequency  $f$  in Hz of your signal in part (3). And what is it in  $\omega = \text{radians/sec}$
6. Draw a sequence of graphs to illustrate what happens as the period of a periodic signal is increased. What happens to the frequency as the period increases.
7. The objective of this problem is to see what happens when we add together pulse trains of frequency  $f_0$  and  $2f_0$ . Given the following two pulse trains of frequency  $f_0 = 1\text{KHz}$  and  $2f_0 = 2\text{KHz}$



- a. Sketch  $x(t) = x_1(t) + x_2(t)$
- b. What is the period of  $x(t)$
- c. Explain in words why  $x(t)$  has the period it does
- d. What is the frequency of  $x(t)$
- e. What would you expect is the frequency of  $x(t)$  equal to a sum of pulse trains at the frequencies  $f_0$ ,  $2f_0$  and  $3f_0$

8. The objective of this problem is to see what happens when we add together sinusoids of frequency  $f_0$  and  $2f_0$ . Suppose in particular that  $x(t) = 2 \cos(2 \cdot 1000t) + 3 \cos(2 \cdot 2000t)$
- Make use of Mathcad to plot  $x(t)$
  - What is the period of  $x(t)$
  - Explain in words why  $x(t)$  has the period it does
  - What is the frequency of  $x(t)$

9. Generalizing on the results of Problems (7) and (8) it can be shown that if  $x(t)$  is a sum of periodic signals at integer multiples of the frequency  $f_0$  as follows

$$f_0, 2f_0, 3f_0, 4f_0, \dots$$

then  $x(t)$  will be periodic of frequency  $f_0$ . Make use of this result to find the period and frequency of  $x(t) = 3 \cos(2 \cdot 200t) + 2 \cos(2 \cdot 400t) + 3 \cos(2 \cdot 800t)$

10. The objective of this problem is to see what happens when we add a constant to a periodic signal
- Sketch a pulse train of frequency  $f_0$
  - Then sketch  $x(t) + K$  with  $K > 0$
  - What is the frequency of your signal  $x(t) + K$
  - Generalize on your result in part (c)
11. Sketch the frequency response of a filter that will "pass" the signal  $\cos(10^5t)$  while "greatly attenuating" the signal  $\cos(10^3t)$ . What is the 3dB frequency of your filter