ECE 209 - AVERAGE POWER - INVESTIGATION 18
AVERAGE POWERS IN RLC CIRCUITS

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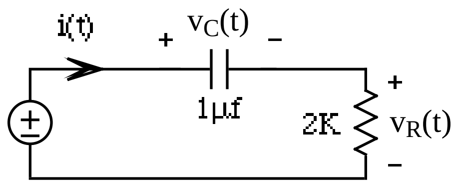
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of the last investigation was to come up with a scheme for calculating the average powers of resistors when the voltages across them are periodic. We developed, in particular, that these average values can be expressed in terms of rms values - the root mean square values of voltages and currents - as follows

$$P_{av} = RI_{rms}^2 = \frac{V_{rms}^2}{R}$$

The objective of this investigation is to calculate the average powers of RLC circuits with sinusoidal inputs. Be sure to remember that complex numbers do **not** in general satisfy $\mathrm{Re}[z_1 z_2] = \mathrm{Re}[z_1]\,\mathrm{Re}[z_2]$ as you do this investigation

1. Let's begin by calculating the powers for the following simple circuit



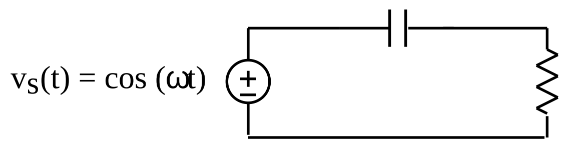
   a. Calculate the steady state values of i(t), $v_C(t)$ and $v_R(t)$ and then make use of them to calculate the powers $p_S(t)$, $p_C(t)$ and $p_R(t)$ - the powers associated with the source, capacitor and resistor
   b. Now make use of your results in part (a) to sketch the powers $p_S(t)$, $p_C(t)$ and $p_R(t)$.
   c. Describe how $p_S(t)$, $p_C(t)$ and $p_R(t)$ are different
   d. Now calculate the average powers P_{av} for the source, capacitor and resistor. You may find the following trig identity helpful:

   $$\cos (x) \cos (y) = 0.5 \cos (x + y) + 0.5 \cos (x - y)$$

   e. Why does the capacitor have zero average power.

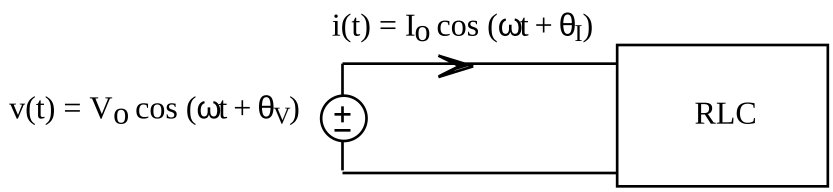
2. Generalize on the result of Problem (1) to show that
   a. The average powers of capacitors in the sinusoidal steady state are always zero.
   b. The average powers of inductors in the sinusoidal steady state are always zero.

3. Pulling together the results of Problems (1) and (2) we see that the average power being delivered to a circuit in the sinusoidal steady state is equal to the average power dissipated by the resistors. And so the average power delivered to a circuit depends on the steady state voltages across the resistors. So - how will increasing the frequency of the source in the following circuit



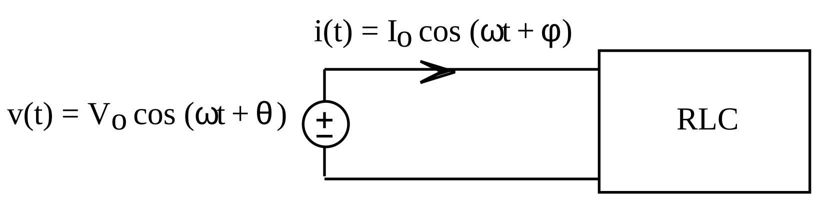
affect the average power being delivered by the source. How can you tell.

4. The objective of this problem is to find expressions for the average power P_{av} being delivered to general RLC circuits as follows



   a. First find P_{av} in terms of I_o and V_o and θ_V and θ_I. Note that you may again the trig identity in Problem (2) to be useful.
   b. Now make use of your result in part (a) to find P_{av} in terms of I_{rms}, V_{rms}, θ_V and θ_I. Note that we refer to $\cos(\theta_V - \theta_I)$ as the **power factor pf**
   c. Sketch the power factor pf as a function of $\theta_V - \theta_I =$ Z(jω) where Z(jω) is the impedance of the circuit. Note that it can be shown that for all ω it's always true that $-\pi/2 \le \theta_V - \theta_I \le \pi/2$. What's the largest possible value of pf

5. Given



with

$$P_{av} = \frac{V_o I_o}{2}\cos(\theta_V - \theta_I) = \frac{1}{2}\,\mathrm{Re}\left[V_o I_o e^{j(\theta_V - \theta_I)}\right] = \frac{1}{2}\,\mathrm{Re}\left[V_o e^{j\theta_V} I_o e^{-j\theta_I}\right] = \frac{1}{2}\,\mathrm{Re}[VI^*]$$

   a. Show that $P_{av} = \frac{1}{2}|I|^2\,\mathrm{Re}[Z]$
   b. Find P_{av} if $I(j1000) = 5e^{j1.2}$ and $Z(j1000) = 100 + j100$

6. The objective of this problem is to find the average power P_{av} of a resistor R with $v_R(t)$ equal to a constant plus a sinusoid at the frequency ω_o. Show, in particular, that when $v_R(t) = c_o + c_1\cos(\omega_o t + \theta_1)$ then $P_{av} = \frac{c_0^2}{R} + \frac{c_1^2}{2R}$

7. Generalizing on the result of Problem (6) it can be shown that if the voltage across a resistor R is a constant plus a sum of sinusoids as follows

$$v_R(t) = c_o + c_1 \cos(\omega_o t + \theta_1) + c_2 \cos(2\omega_o t + \theta_2) + c_3 \cos(3\omega_o t + \theta_3) + \cdots$$

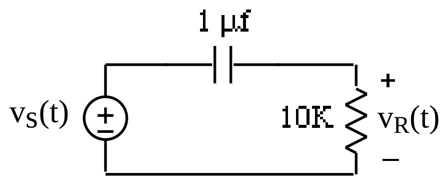
at the frequencies ω_o, $2\omega_o$, $3\omega_o$, . . . then the average power P_{av} of the resistor is the sum of the powers of the individual sinusoids as follows

$$P_{av} = \frac{c_o^2}{R} + \frac{c_1^2}{2R} + \frac{c_2^2}{2R} + \cdots = \sum_{k=0}^{\infty} P_k \quad \text{where} \quad P_0 = \frac{c_o^2}{R},\ P_1 = \frac{c_1^2}{2R},\ P_2 = \frac{c_2^2}{2R},\ \ldots$$

Make use of this result to find the average power P_{av} of a 1K resistor with voltage

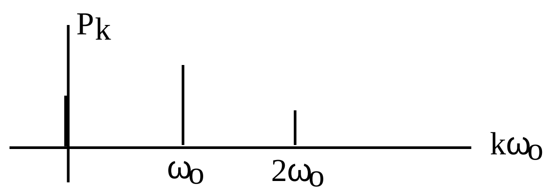
$$v_R(t) = 2 + 3\cos(1000t) + 2\cos(2000t + 1.2)$$

8. Now suppose we have the following circuit

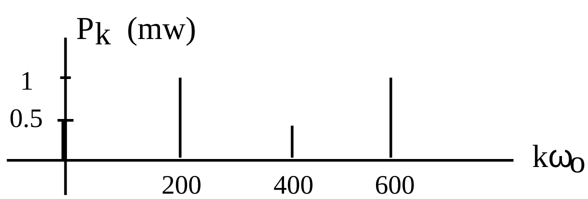


with input $v_S(t) = 2 + 3\cos(100t + 1.2) + 1.5\cos(200t - 0.5)$

   a. Find the average power being delivered to the resistor by the source $v_S(t)$
   b. Then make a **power spectral plot** for the resistor like the following

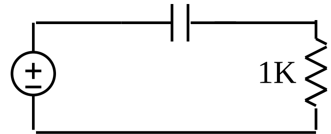


9. Find the average power of a 1K resistor with power spectral plot as follows



10. Find the average power P_{av} being delivered to the following circuit

$v_S(t) = 5\cos(1000t + 1.2) + 3\cos(2000t)$



with transfer function

$$G(j\omega) = \frac{1000}{1000 + j\omega}$$