ECE 209 - AVERAGE POWER - INVESTIGATION 17
AVERAGE POWERS OF PERIODIC SIGNALS

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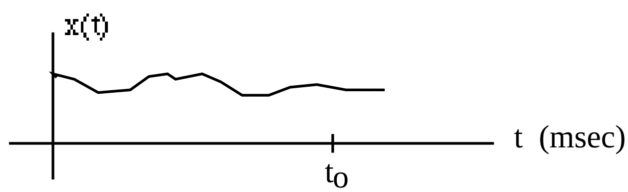
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

When it comes to power, there are two important quantities of interest. The first quantity is peak power because it's peak powers that stress power sources and burn out circuit components. The second quantity is average power because with average powers we can easily calculate total energy requirements. The objective of this investigation is to come up with a scheme for calculating the average powers of resistors when the voltages across them are periodic signals like pulse trains and sinusoids.

1. Let us begin with the following nonperiodic signal



   How you would calculate the average value of x(t) from $t = 0$ to $t = t_o$

2. From Problem (1) we know that the average of a signal x(t) from time $t = 0$ to time t is given by

$$x_{ave}(t) = \frac{1}{t}\int_0^t x(t)\, dt$$

   The objective of this problem is to calculate $x_{ave}(t)$ for the following periodic signal

$$x(t) = 4 + 2\cos(2\pi 1000t)$$

   and then see what happens as t increases.
   a. First sketch x(t)
   b. Then calculate $x_{ave}(t)$
   c. Now sketch $x_{ave}(t)$ for $0 \le t \le 5$ msec. Describe what's happening.
   d. From part (c) we see that as t gets larger, $x_{ave}(t)$ is getting closer and closer to a constant value - a value we define to be the **average X_{av}** of x(t). Formally we have

$$X_{av} \equiv \lim_{t\to\infty} x_{ave}(t) = \lim_{t\to\infty} \frac{1}{t}\int_0^t x(t)dt$$

   Make use of your graph to estimate X_{av} for x(t).
   e. Find the period T of x(t)
   f. Now find the average of x(t) over one period T
   g. How are your averages in parts (d) and (f) related.
   h. Why do you think your averages in parts (d) and (f) are related the way they are.

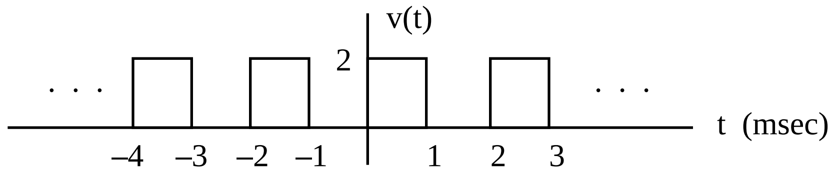
3. Generalizing on the result of Problem (2) it can be shown that the average value of **any**

periodic signal over a "large" amount of time - a "whole bunch" of cycles - is equal to its average value over one period as follows

$$X_{av} \equiv \lim_{t \to \infty} x_{ave}(t) = \frac{1}{T}\int_T x(t)dt = \text{ Average of x(t) over one period}$$

Memorize this result.

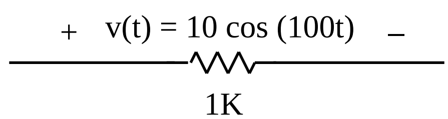
a. Make use of this result to find the average power P_{av} of a 1K resistor with voltage



Be sure to first sketch a graph of p(t) before doing any calculations

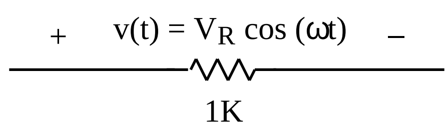
b. Make use of your result in part (a) to calculate the total energy dissipated by the resistor in one minute

4. The objective of this problem is to find an average power P_{av} just like in Problem (3) but this time for when v(t) is a periodic sinusoid. Suppose in particular that v(t) = 10 cos (100t) is the voltage across the following 1K resistor



a. First explain in words why the average of a sinusoid over a period is zero. Draw a picture to illustrate
b. Now find and sketch the power $p_R(t) = v_R(t)\, i_R(t)$ being delivered to the resistor as a function of time. Remember that $\cos^2 x = 0.5 + 0.5 \cos 2x$
c. Then find P_{av}
d. What is the total energy dissipated by the resistor in one minute.

5. Now suppose we have the more general sinusoidal voltage across the resistor



a. Find and sketch the power p(t) as a function of time
b. Find P_{av}. How does your result depend on frequency. Explain why

6. The objective of this problem is to define what we mean by root mean square (rms) values of periodic signals. Beginning with the fact that the power for a resistor of value R is $p(t) = v^2(t)/R$, we have

$$\text{ave}\,[p(t)] = \text{ave}\left[\frac{v^2(t)}{R}\right] = \frac{ave\,[v^2(t)]}{R} = \frac{\left(\sqrt{ave\,[v^2(t)]}\right)^2}{R} = \frac{V_{rms}^2}{R}$$

with V_{rms} defined as follows

$$V_{rms} \equiv \sqrt{ave\,[v^2(t)]} \equiv \textbf{rms} \text{ voltage } = \textbf{ root} \text{ of the } \textbf{mean} \text{ of the } \textbf{square} \text{ of the voltage}$$

a. Why do we call V_{rms} the root of the mean of the square of the voltage
b. Why would you expect that we introduce rms values. What, in particular, are the similarities between our expression for the average power of a periodic signal

$$ave[p(t)] = \frac{ave[v^2(t)]}{R} = \frac{V_{rms}^2}{R}$$

in terms of rms values and the expression for power in DC resistor circuits with constant inputs
c. Find V_{rms} for a sinusoid $v_R(t) = V_R \cos(\omega t)$
d. Derive an expression for P_{av} in terms of I_{rms} and R
e. Find V_{rms} for the pulse train of Problem (4)