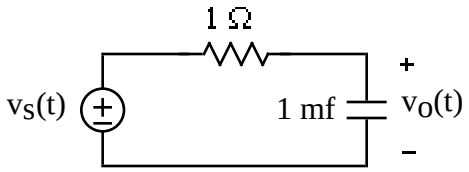
ECE 209 - INTRODUCTION TO FILTERS - INVESTIGATION 16
MAGNITUDE AND FREQUENCY SCALING

FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to see how magnitude and frequency scaling affect the frequency responses of circuits.

1. The objective of this problem is to introduce magnitude scaling. Given the following RC circuit



   a. Find the voltage transfer function of this *original* circuit

$$G_{orig}(j\omega) = \frac{V_o(j\omega)}{V_s}$$

   b. Now suppose we **magnitude scale** the circuit by $R_O = 1000$ - replace every resistor, capacitor and inductor by a new circuit element with impedance $R_O = 1000$ times as much so that

$$Z_{new}(j\omega) = R_o Z_{orig}(j\omega) = 1000 Z_{orig}(j\omega)$$

   Draw the new magnitude scaled phasor circuit

   c. Find the transfer function

$$G_{new}(j\omega) = \frac{V_o(j\omega)}{V_s}$$

   of the new magnitude scaled circuit.

   d. How is the transfer function of the magnitude scaled circuit related to that of the original circuit.

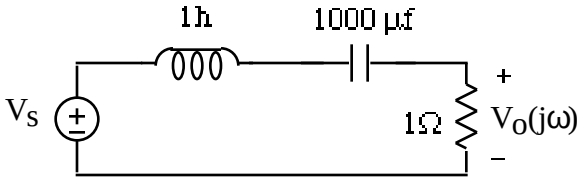
   e. Why do you think $G_{orig}(j\omega)$ and $G_{new}(j\omega)$ are related the way they are

2. Generalizing on the result of Problem (1) it can be shown that voltage and current transfer functions as follows

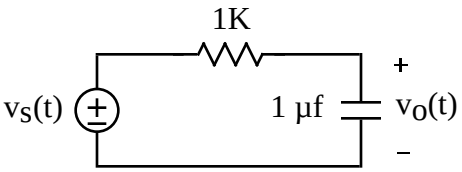
$$G(j\omega) = \frac{V_o(j\omega)}{V_s} \quad \text{and} \quad G(j\omega) = \frac{I_o(j\omega)}{I_s}$$

   are not affected by magnitude scaling - they stay the same. Find the new values of resistors R, capacitors C and inductors L after they are magnitude scaled by R_O. Put your results in a Table.

3. Find and draw the following circuit after it is magnitude scaled by $R_o = 1000$



4. The objective of this problem is to introduce frequency scaling. Given the following RC circuit



   a. Find and sketch the magnitude of the transfer function of this *original* circuit

$$G_{orig}(j\omega) = \frac{V_o(j\omega)}{V_s}$$

   with ω plotted on a log scale.

   b. Now suppose we **frequency scale** the circuit by $\omega_o = 10$ - replace every resistor, capacitor and inductor by a new circuit element with impedance given by

$$Z_{new}(j\omega) = Z_{orig}\left(j\frac{\omega}{\omega_o}\right) = Z_{orig}\left(j\frac{\omega}{10}\right)$$

   Draw the new frequency scaled phasor circuit

   c. Find and sketch the magnitude of the transfer function

$$G_{new}(j\omega) = \frac{V_o(j\omega)}{V_s}$$

   of your new frequency scaled circuit.

   d. How is the magnitude of the transfer function of the frequency scaled circuit related to that of the original circuit.

   e. Why do you think $G_{orig}(j\omega)$ and $G_{new}(j\omega)$ are related the way they are

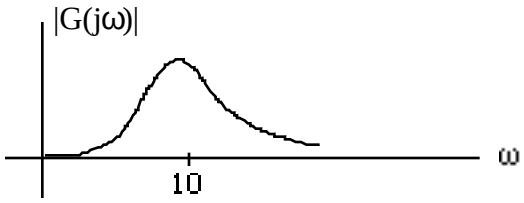
5. Generalizing on the result of Problem (4) it can be shown that if we frequency scale a circuit by ω_o then the gains of the new and scaled circuit will be related by

$$G_{new}(j\omega) = G_{orig}\left(j\frac{\omega}{\omega_o}\right)$$

   or equivalently

$$G_{orig}(j\omega) = G_{new}\left(j\omega\omega_o\right)$$

   Make use of this result to sketch the frequency response of a circuit with the following frequency response



after it is frequency scaled by $\omega_o = 100$.

6. Find the new values of resistors R, capacitors C and inductors L after they are frequency scaled by ω_o. Put your results in a Table.

7. Find and draw the following circuit after it is frequency scaled by $\omega_o = 1000$ and magnitude scaled by $R_o = 1000$