ECE 209 - INTRODUCTION TO FILTERS - INVESTIGATION 15
BANDPASS FILTERS

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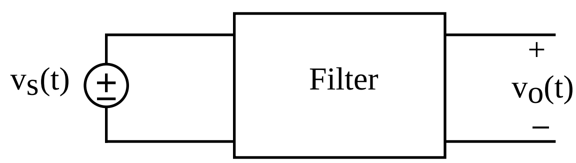
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last investigation we introduced how to analyze and design lowpass and highpass filters. The objective of this investigation is to introduce how to analyze and design bandpass filters. But first we begin with a lowpass review problem.

1. Design a first order lowpass RC filter with input $v_S(t) = 2\cos(2\text{x}10^3 t) + 3\cos(3\text{x}10^5 t)$ that will "pass" the lower frequency sinusoid but filter out the higher frequency one. Do calculations and plots to see how good a job your filter is doing.

2. Suppose we would like to build a filter as follows



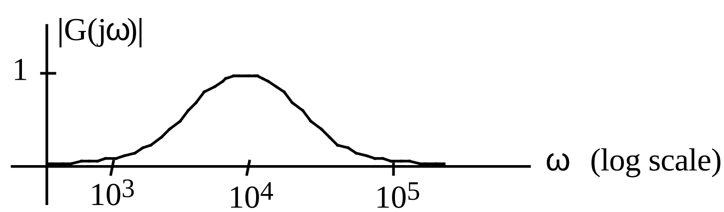
so that when

$$v_S(t) = 2\cos(10^3 t) + 3\cos(10^4 t) + 2\cos(10^5 t)$$

then the steady state output will be approximately equal to

$$v_O(t) = 3\cos(10^4 t)$$

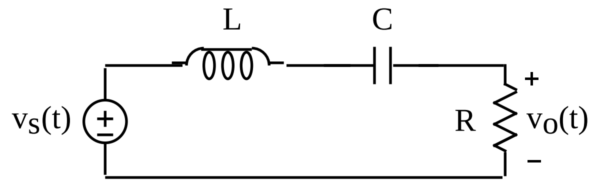
The first step in the design of such a filter is to sketch a graph of the desired bandpass frequency response as follows



and then quantify it with inequalities like

$$G(j10^3) \leq 0.1 \qquad G(j10^4) \approx 1 \qquad G(j10^5) \leq 0.1$$

The next step is to find a circuit with a bandpass frequency response that meets the specs. A good place to start is with one of our second order resonant circuits as follows



a. First write $G(j\omega)$ as function of ω_p and Q_p
b. Choose values for ω_p and the 3dB Bandwidth so that $G(j10^3) \le 0.1$, $G(j10^4) \approx 1$ and $G(j10^5) \le 0.1$. Then write the corresponding transfer function $G(j\omega)$ in terms of ω_p and Q_p
c. Make use of your transfer function in part (a) to calculate your values for $\left|G(j10^3)\right|$, $\left|G(j10^4)\right|$ and $\left|G(j10^5)\right|$
d. Draw the spectral plots (amplitudes only) of $v_S(t)$ and your $v_O(t)$.
e. Find and plot the steady state response of your circuit.
f. Describe how good a job your transfer function does of meeting the specs - of passing the sinusoid at the frequency $\omega = 10^3$ and filtering out the sinsuoids at the frequencies $\omega = 10^3$ and $\omega = 10^5$
g. Find practical values of R, L and C to realize your transfer function.

3. Second order circuits like those in Problem (1) work fine when the differences between the frequencies of the sinusoids in the passbands and stopbands aren't too close. But when the frequencies are closer together we need filters with narrower **transition regions** - we need filters that transition from the passband to stopband over a narrower range of frequencies. To accomplish this we need higher order filters.

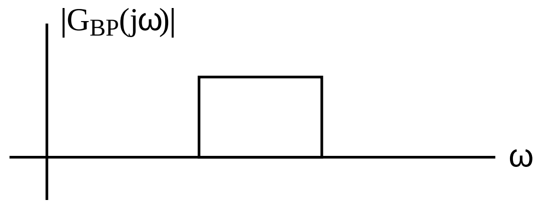
   To see how increasing the order of a bandpass filter can affect its frequency response let us consider the following **Maximally Flat** bandpass transfer functions of second and fourth order filters as follows

$$G_2(j\omega) = \frac{j\omega}{(j\omega)^2 + j\omega + 1}$$

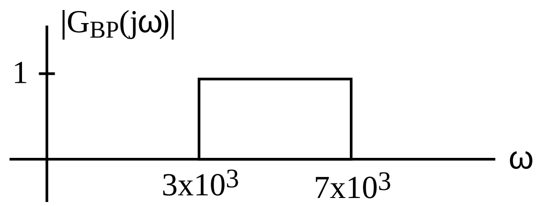
$$G_4(j\omega) = \left(\frac{0.69 j\omega}{(j\omega)^2 + 0.46(j\omega) + 0.48}\right)\left(\frac{1.44 j\omega}{(j\omega)^2 + 0.96(j\omega) + 2.08}\right)$$

   a. Now plot the magnitudes of transfer functions on the same graph over the frequency range $\omega = 0.1$ to $\omega = 10$ with ω plotted on a log scale.
   b. Describe what's happening to the magnitudes of the transfer functions as the order increases.

4. As the order of the bandpass filters in Problem (3) get higher and higher their frequency responses can be made closer and closer to the ideal bandpass response as follows



Find the steady state response of the following ideal bandpass filter



to the input $v_S(t) = \cos(10^3 t) + 2\cos(5\text{x}10^3 t) + 2\cos(10^4 t)$. Be sure to draw spectral plots of the input and output.

5. Sketch the frequency response of an ideal bandpass filter with input

$$v_S(t) = \cos(10^3 t) + \cos(2\text{x}10^3 t) + \cos(5\text{x}10^3 t)$$

and steady state output

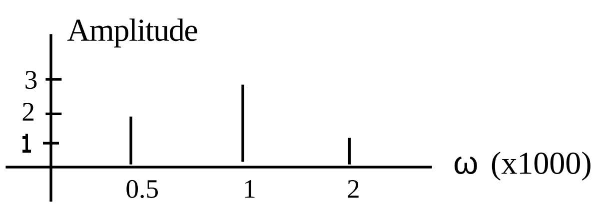
$$v_O(t) = 2\cos(2\text{x}10^3 t)$$

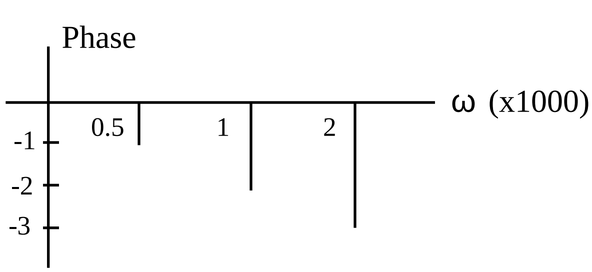
Be sure to draw spectral plots of the input and output.

6. Find the steady state response $v_O(t)$ of a circuit with frequency response as follows

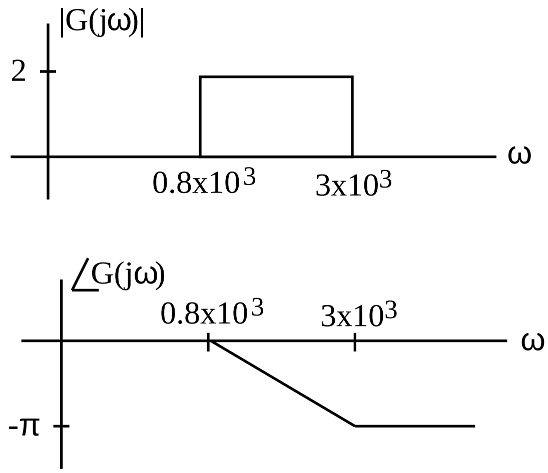
$$G(j\omega) = \frac{2000(j\omega)}{(j\omega)^2 + 1000(j\omega) + 10^6}$$

when the input $v_S(t)$ has the following spectral plot





7. Find the steady state response $v_O(t)$ of a circuit with frequency response as follows



and the same input as in Problem (6).