# ECE 209 - INTRODUCTION TO FILTERS - INVESTIGATION 14
INTRODUCTION TO LOWPASS AND HIGHPASS FILTERS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last investigation we showed how superposition can be used to find the steady state responses of linear circuits like the following
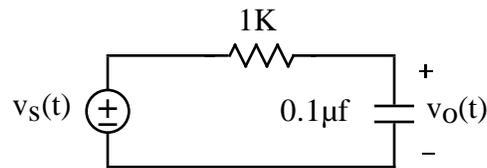
when their inputs are equal to sums of sinusoids like $v_S(t) = 3\cos(100t) + 2\cos(300t)$. The objective of this investigation is to show how we can make use of superposition to analyze **filters** - very useful circuits that allow only desired sinusoids to reach the output.

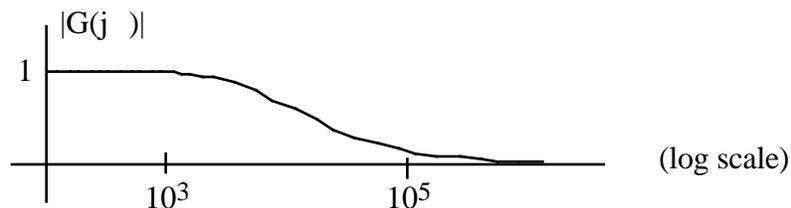1. Suppose we would like to build a filter as follows
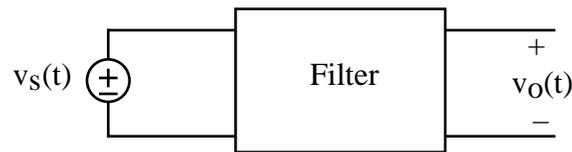
so that when the input $v_S(t)$ is equal to the sum

$$v_S(t) = 2\cos(10^3 t) + 2\cos(10^5 t)$$

then the steady state output $v_O(t)$ will be approximately equal to

$$v_O(t) = 2\cos(10^3 t)$$

The first step in the design of the filter is to sketch a graph of the desired lowpass frequency response as follows

and then quantify the contraints with inequalities like the following

$$0.9 \le \left| G(j10^3) \right| \le 1 \quad \text{and} \quad \left| G(j10^5) \right| \le 0.1$$

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We call these inequalities the filter's specs. Note that we say that $\omega = 10^3$ rad/sec is in the **passband** of the filter and $\omega = 10^5$ rad/sec is in the **stopband**.

The next step is to find a circuit that meets the specs. One that does a pretty good job is the following first order RC circuit
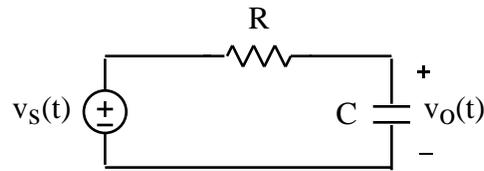
with transfer function

$$G(j\omega) = \frac{V_o(j\omega)}{V_s} = \frac{\dfrac{1}{RC}}{j\omega + \dfrac{1}{RC}} = \frac{\omega_{3dB}}{j\omega + \omega_{3dB}}$$

    a. Choose a 3dB frequency ω_{3dB} to meet the spec. And then write the corresponding transfer function $G(j\omega)$

    b. Compare your values for $\left|G(j10^3)\right|$ and $\left|G(j10^5)\right|$ with the desired values

    c. Find the steady state response of $v_o(t)$ of your circuit

    d. Draw the spectral plots (amplitudes only) of $v_s(t)$ and your $v_o(t)$

    e. Use Mathcad to obtain a graph of $v_o(t)$. Make your graph over a long enough time to see at least one cycle of $\cos(10^3 t)$ and Δt small enough to plot at least 10 points/cycle of the sinusoid $\cos(10^5 t)$

    f. Describe how good a job does your circuit do of passing the sinusoid at the frequency $\omega = 10^3$ and filtering out the sinsuoid at the frequency $\omega = 10^5$

    g. Find practical values of R and C for your circuit in part (a).

2. Design a first order highpass RC filter with input $v_s(t) = 2\cos(2x10^3 t) + 3\cos(3x10^5 t)$ that will pass the higher frequency sinusoid but filter out the lower frequency one. Do calculations and plots like you did in Problem (1) to verify that your circuit will do the job.

3. First order circuits like those in Problems (1) and (2) work fine when the differences between the frequencies of the sinusoids in the passbands and stopbands are far apart. But when the frequencies are closer together we need filters with frequency responses that transition from the passband to the stopband over a narrower range of frequencies - filters that have narrower **transition regions**. To accomplish this we need higher order filters.

To see how increasing the order of a filter can affect its frequency response let us consider the following lowpass transfer functions of first, second and third order **Butterworth** or **Maximally Flat** filters

$$G_1(j\omega) = \frac{1}{j\omega + 1} \qquad G_2(j\omega) = \frac{1}{(j\omega)^2 + 1.4 j\omega + 1}$$

$$G_3(j\omega) = \frac{1}{((j\omega)^2 + j\omega + 1)(j\omega + 1)}$$

a. Make use of the fact that each of these lowpass transfer functions is maximum at $\omega = 0$ to verify - not derive - that all three transfer functions have 3dB frequencies at 1 rad/sec. We say such lowpass filters have *normalized* transfer functions

b. Now use Mathcad to plot the magnitudes of all three of these transfer functions on the same graph over the frequency range $\omega = 0.1$ to $\omega = 10$ with ω plotted on a log scale.

c. Describe what's happening to the magnitudes of the transfer functions as the order increases.

4. As the orders of our lowpass and highpass filters get higher and higher their frequency responses get closer and closer to **ideal** lowpass and highpass frequency responses as follows
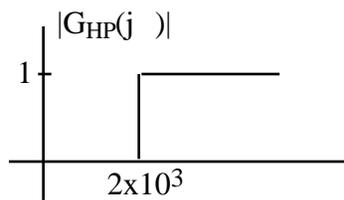

a. Find the steady state response of the following ideal lowpass filter
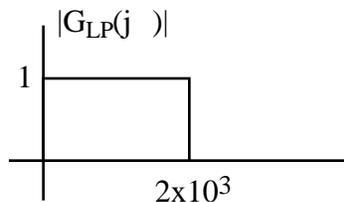
to $v_S(t) = \cos(10^3 t) + 2\cos(3 \times 10^3 t)$. Draw spectral plots for the input and output. And then use Mathcad to plot $v_S(t)$ and $v_O(t)$

b. Find the steady state response of the following ideal highpass filter

to $v_S(t) = \cos(10^3 t) + 2\cos(3 \times 10^3 t)$. Draw spectral plots for the input and output. And then use Mathcad to plot $v_S(t)$ and $v_O(t)$

5. Sketch the frequency response of an ideal lowpass filter with input

$$v_S(t) = \cos(10^3 t) + \cos(2 \times 10^3 t) + \cos(5 \times 10^3 t)$$

and steady state output

$$v_O(t) = 2\cos(10^3 t) + 2\cos(2 \times 10^3 t)$$