ECE 209 - INTRODUCTION TO FILTERS - INVESTIGATION 13
SUPERPOSITION OF STEADY STATE RESPONSES

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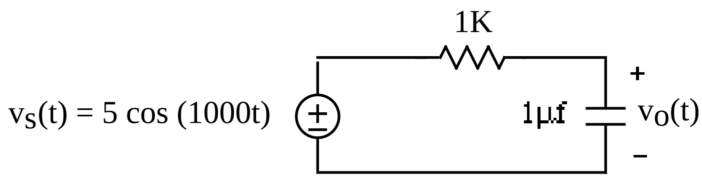
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

Up to now we've only been calculating the sinusoidal steady state responses of circuits with single sinusoidal sources like the following



But many circuits have either more than one sinusoidal source or else have a single source equal to a sum of sinusoids. The objective of this investigation is to show how superposition can be used to find the steady state responses of these circuits.

1. The objective of this first problem is to review a basic result from linear differential equations on forced responses. Show that if $x_{f1}(t)$ is the forced response of

$$x' + 100x = f_1(t)$$

and $x_{f2}(t)$ is the forced response of

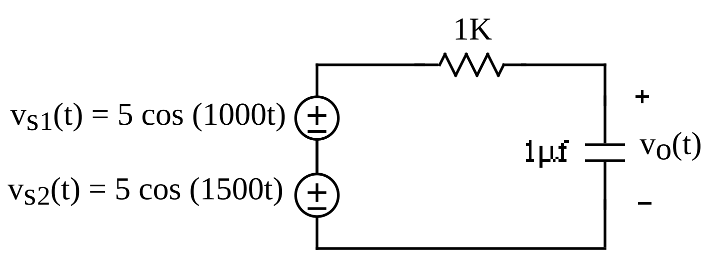
$$x' + 100x = f_2(t)$$

then $x_f(t) = x_{f1}(t) + x_{f2}(t)$ is the forced response of the differential equation

$$x' + 100x = f_1(t) + f_2(t)$$

Hint - Substitute $x_f(t) = x_{f1}(t) + x_{f2}(t)$ into $x_f' + 100x_f$ and show that it equals $f_1(t) + f_2(t)$. **Memorize** this result. It tells us that we can use **superposition** to find the forced response of a circuit to a sum of inputs by simply finding the forced response to each one separately and then add them up.

2. Make use of the superposition result from Problem (1) to find the steady state response of the following circuit

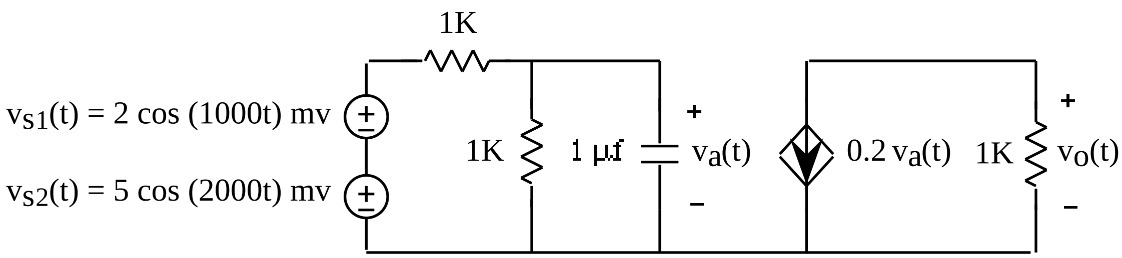


by adding the forced response of the circuit to $v_{S1}(t)$ when $v_{S2}(t) = 0$ to the forced response of the circuit to $v_{S2}(t)$ when $v_{S1}(t) = 0$

a. First draw and then analyze the phasor circuit to find the sinusoidal steady state response $v_{01}(t)$ to $v_{S1}(t)$ when $v_{S2}(t) = 0$

b. Then draw and analyze the phasor circuit to find the sinusoidal steady state response $v_{02}(t)$ to $v_{s2}(t)$ when $v_{s1}(t) = 0$
c. Now make use of superposition to find the forced response $v_o(t) = v_{01}(t) + v_{02}(t)$ when both $v_{s1}(t)$ and $v_{s2}(t)$ are in the circuit.
d. Make use of Mathcad or an equivalent to get plots of $v_s(t) = v_{s1}(t) + v_{s2}(t)$ and the steady state $v_o(t)$

3. Make use of superposition to find the steady state response of the following circuit

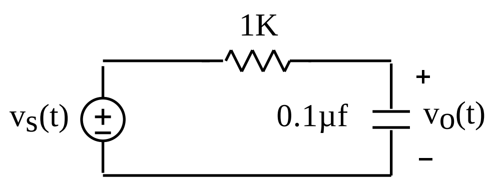


4. Find the steady state response of a circuit with the following transfer function

$$G(j\omega) = \frac{3j\omega}{j\omega + 1500}$$

to the input $v_s(t) = \cos(1500t) + 2\cos(2000t + \pi/6)$

5. Find the steady state response of the following circuit

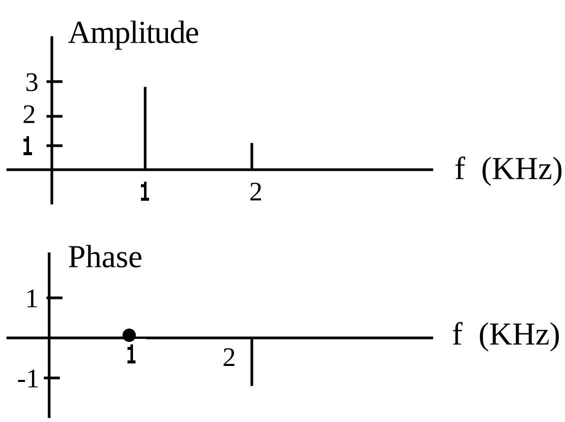


to the input $v_s(t) = 3\cos(10^4 t) + 2\cos(2\text{x}10^4 t)$

6. One good way to get a picture of what's going on in circuits with more than one sinusoidal source is with spectral plots. A **spectral plot** of a signal like v(t) as follows

$$v(t) = 3\cos(2\pi 1000t) + 2\cos(2\pi 2000t - 1.2)$$

is simply a plot of the amplitudes and phases of the individual sinusoids as follows



Now suppose we have a circuit with input

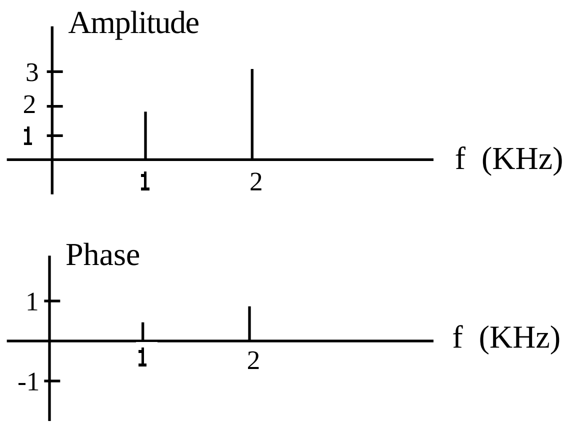
$$v_S(t) = 2\cos(2\pi 2000t - 0.5) + \cos(2\pi 3000t + 1)$$

and steady state response

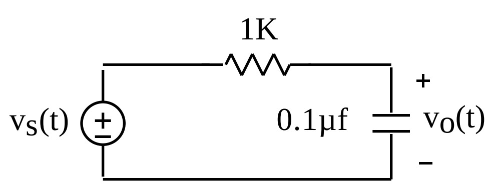
$$v_O(t) = 4\cos(2\pi 2000t - 1) + 2\cos(2\pi 3000t + 0.5)$$

a. Sketch the spectral plot of $v_S(t)$
b. And then sketch the spectral plot of the steady state output $v_O(t)$

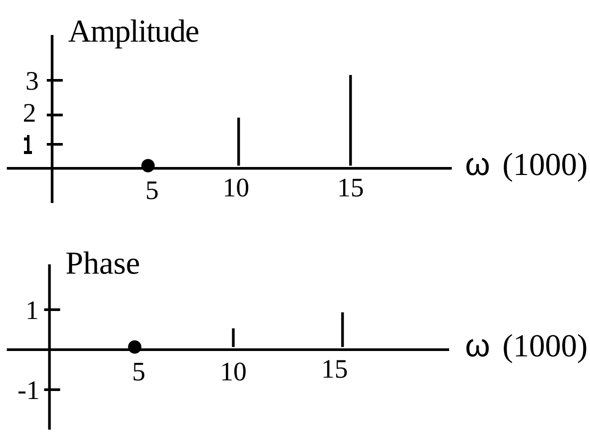
7. Find v(t) for the signal with the following spectral plot



8. Given the following circuit

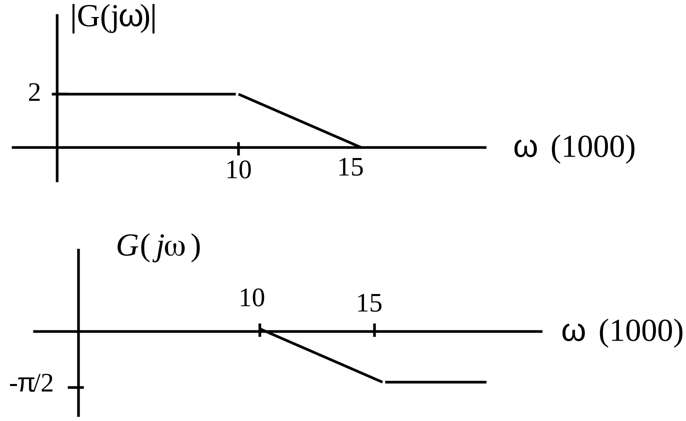


a. Find the steady state response of $v_O(t)$ to the input with the following spectral plot

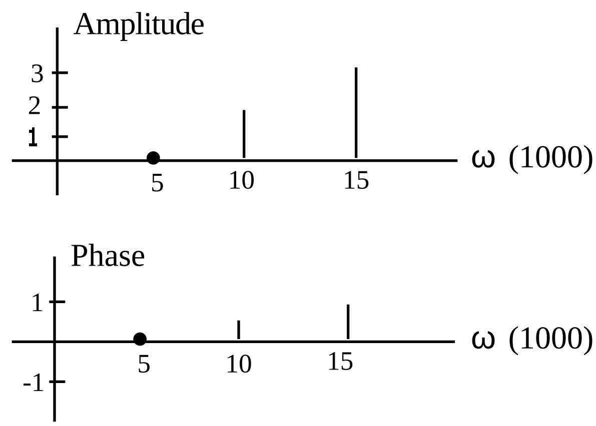


b. Sketch the spectral plot of the steady state $v_O(t)$

9. Find the steady state response of a circuit with the following frequency response



to the input $v_S(t)$ with spectral plot



10. Find the steady state response of a circuit with the transfer function

$$G(j\omega) = \frac{j\omega}{j\omega + 1000}$$

to the same input $v_S(t)$ as in Problem (9)