ECE 209 - SECOND ORDER CIRCUITS - INV 12
FREQUENCY RESPONSES OF GENERAL 2ND ORDER CIRCUITS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The main objective of this investigation is to show how the parameters ω_p and Q_p characterize the frequency responses of general second order circuits.

1. The transfer functions of general second order bandpass circuits are often written in the form

$$G(j\omega) = \frac{K\dfrac{\omega_p}{Q_p}(j\omega)}{(j\omega)^2 + \dfrac{\omega_p}{Q_p}(j\omega) + \omega_p^2}$$

where ω_p and Q_p are constants that characterize what $G(j\omega)$ looks like

a. Verify that ω_p is the resonant frequency of this transfer function. Hint - divide both the numerator and denominator by $j\omega$ to get the transfer function in the form of Problem (7) in Investigation 10.

b. Find $G(j\omega_p)$ equal to the gain at resonance when $\omega = \omega_p$

c. Make use of Mathcad to plot the magnitudes of the following transfer functions on the same graph. Plot ω on a log scale.

$|G_1(j\omega)|$ with $K = 1$, $Q_p = 1$ and $\omega_p = 10^3$

$|G_2(j\omega)|$ with $K = 1$, $Q_p = 1$ and $\omega_p = 10^4$

d. Make use of your graphs in part (c) to describe how the value of ω_p affects the frequency response.

e. Make use of Mathcad to plot the magnitudes of the following transfer functions on the same graph. Plot ω on a log scale.

$|G_1(j\omega)|$ with $K = 1$, $\omega_p = 10^3$ and $Q_p = 0.5$

$|G_2(j\omega)|$ with $K = 1$, $\omega_p = 10^3$ and $Q_p = 2$

f. Make use of your graphs in part (e) to describe how the value of Q_p affects the frequency response.

g. Make use of the fact that the 3dB frequencies ω_1 and ω_2 of a 2nd order bandpass circuit are

$$\omega_1 = -\frac{\omega_p}{2Q_p} + \frac{\omega_p}{2Q_p}\sqrt{4Q_p^2 + 1} \qquad \omega_2 = \frac{\omega_p}{2Q_p} + \frac{\omega_p}{2Q_p}\sqrt{4Q_p^2 + 1}$$

to find an expression for the 3dB bandwidth in terms of ω_p and Q_p

h. How does increasing Q_p affect the 3dB bandwidth of a second order bandpass circuit.

2. Sketch $|G(j\omega)|$ as a function of ω plotted on a log scale for a second order bandpass circuit with $\omega_p = 10^5$ and $Q_p = 4$. Explain how you got your result.

3. Given the following transfer function of a second order bandpass circuit

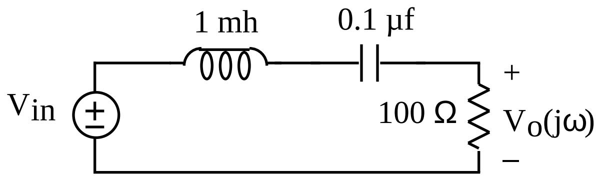
$$G(j\omega) = \frac{10^3(j\omega)}{(j\omega)^2 + 2x10^3(j\omega) + 10^6}$$

a. Find ω_p and Q_p

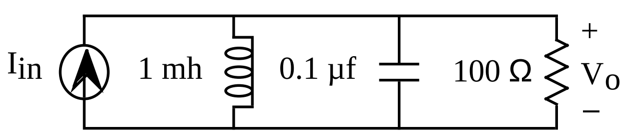
b. Sketch $|G(j\omega)|$ as a function of ω plotted on a log scale. Explain how you got your result.

4. Find ω_p and Q_p for the following circuits

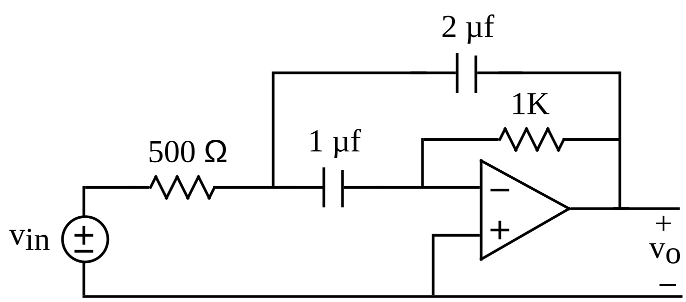
a.



b.



5. The objective of this problem is to demonstrate that we don't need inductors to build second order bandpass circuits - all we need is resistors, capacitors and op amps as in the following circuit. Note that we refer to such circuits as **RC-Active filters**.



a. Make use of node equations to verify that this is a second order bandpass circuit with the following voltage transfer function

$$G(j\omega) = -\frac{10^3(j\omega)}{(j\omega)^2 + 1.5x10^3(j\omega) + 10^6}$$

Always be sure to write transfer functions $G(j\omega)$ as rational polynomials in $j\omega$ like in this problem.

b. Sketch $|G(j\omega)|$ as a function of ω on a log scale.

6. Up to now we've just been looking at the transfer functions of second order bandpass transfer

functions. But second order transfer functions can also be lowpass of the form

$$G_{LP}(j\omega) = \frac{K}{(j\omega)^2 + \frac{\omega_p}{Q_p}(j\omega) + \omega_p^2}$$

as well as highpass of the form

$$G_{HP}(j\omega) = \frac{K(j\omega)^2}{(j\omega)^2 + \frac{\omega_p}{Q_p}(j\omega) + \omega_p^2}$$

a. Make use of Mathcad to obtain graphs of the magnitude of the lowpass transfer function

$$G_{LP}(j\omega) = \frac{10^6}{(j\omega)^2 + \frac{10^3}{Q_p}(j\omega) + 10^6}$$

as a function of ω on a log scale for $Q_p = 0.5$, $Q_p = 0.7$ and $Q_p = 1$ all on the same graph.

b. Describe what happens to $|G_{LP}(j\omega)|$ as Q_p increases.

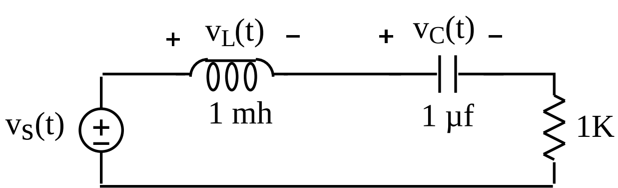
c. Make use of Mathcad to obtain graphs of the magnitude of the highpass transfer function

$$G_{HP}(j\omega) = \frac{(j\omega)^2}{(j\omega)^2 + \frac{10^3}{Q_p}(j\omega) + 10^6}$$

as a function of ω on a log scale for $Q_p = 0.5$, $Q_p = 0.7$ and $Q_p = 1$ all on the same graph.

d. Describe what happens to $|G_{HP}(j\omega)|$ as Q_p increases.

7. The objective of this problem is to illustrate how lowpass and highpass transfer functions can be obtained from RLC circuits. Given the following series RLC circuit



a. Verify that $G_L(j\omega) = V_L(j\omega)/V_S$ is a highpass transfer function

b. Verify that $G_C(j\omega) = V_C(j\omega)/V_S$ is a lowpass transfer function