ECE 209 - SECOND ORDER CIRCUITS - INV 11
FREQUENCY RESPONSES OF PARALLEL RLC CIRCUITS

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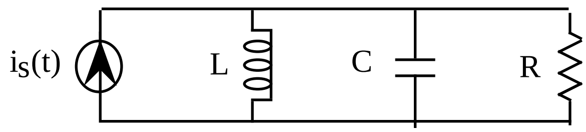
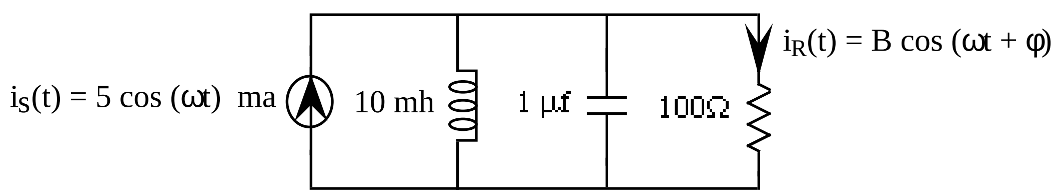
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to analyze parallel RLC resonant circuits as follows



just like we did for series RLC circuits in the last investigation.

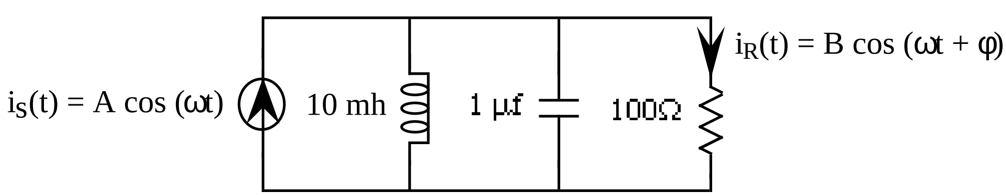
1. The objective of this problem is to see how the amplitude B of the sinusoidal steady state response of $i_R(t)$ in the following parallel RLC circuit



varies as a function of frequency ω.

   a. Find the steady state response of $i_R(t)$ when the input is a constant - a sinusoid of frequency $\omega = 0$.
   b. What would you expect happens to the amplitude of $i_R(t)$ as ω increases from 0. Test your result by calculating B at $\omega = 500$ rad/sec
   c. Find the steady state response of $i_R(t)$ when $\omega = \infty$.
   d. Make use of your results in parts (a)-(c) to sketch $|I_R(j\omega)|$ as a function of frequency ω.

2. The objective of this problem is to actually calculate and sketch the transfer function



of the bandpass circuit in Problem (1) as given by

$$G(j\omega) = \frac{I_R(j\omega)}{I_s}$$

   a. Find $G(j\omega) = I_R(j\omega)/I_S$

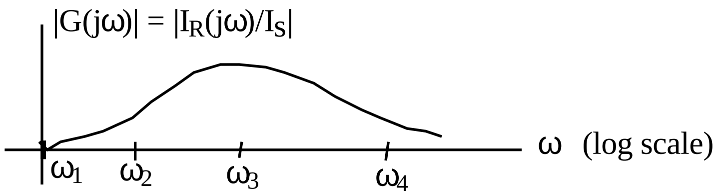
b. Use Mathcad to obtain a graph of $|G(j\omega)|$ as a function of ω on a log scale
c. Describe your graph in part (b)
d. Reconcile any differences between your graphs in part (b) of this problem and part (d) in Problem (1)

3. Find the sinusoidal steady state response of a second order parallel RLC circuit with the following transfer function

$$G(j\omega) = \frac{I_R(j\omega)}{I_s} = \frac{1000(j\omega)}{(j\omega)^2 + 1000(j\omega) + 10^6}$$

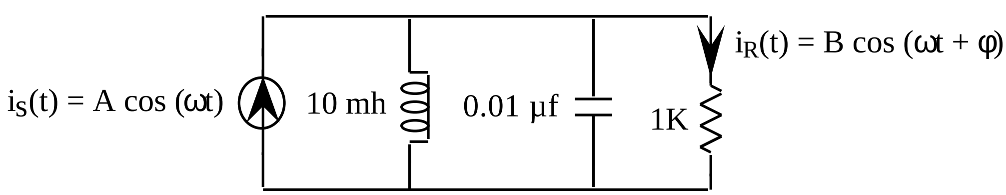
to the input $i_S(t) = 5 \cos(1000t)$ ma

4. The objective of this problem is to sketch graphs of what we would see on a scope as we increase the frequency in a parallel RLC resonant circuit like the one in Problems (1) and (2) with frequency responses as follows



a. Sketch $i_S(t)$ and the sinusoidal steady state response $i_R(t)$ at each of the frequencies $\omega = \omega_1,\ \omega_2,\ \omega_3,\ \omega_4$. Put all sketches on separate graphs.
b. Describe what's happening to the steady state response of $i_R(t)$ as ω increases from 0

5. The objective of this problem is to graphically find the resonance frequency ω_p and 3dB bandwidth of the following parallel RLC resonant circuit

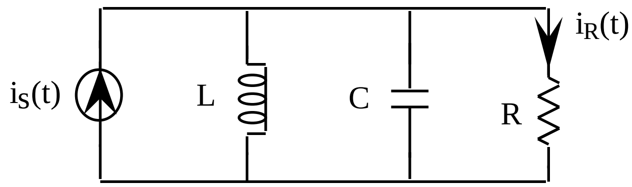


with bandpass transfer function as follows

$$G(j\omega) = \frac{I_R(j\omega)}{I_s} = \frac{10^5(j\omega)}{(j\omega)^2 + 10^5(j\omega) + 10^{10}}$$

a. Use Mathcad or an equivalent to obtain a graph of $|G(j\omega)| = |I_R(j\omega)/I_S|$ versus ω on a log scale.
b. Make use of your graph in part (a) to find the resonance frequency ω_p
c. Make use of your graph in part (a) to find the lower and upper 3dB frequencies $\omega_{\ell 3dB}$ and ω_{u3dB} of $G(j\omega)$
d. Make use of your graph in part (c) to find the 3dB Bandwidth = $\omega_{u3dB} - \omega_{\ell 3dB}$

6. The objective of this problem is to analyze a general second order parallel RLC resonant circuit as given by



   a. Verify that the current transfer function is

$$G(j\omega) = \frac{I_R(j\omega)}{I_s} = \frac{1/R}{1/R + j\omega C + \frac{1}{j\omega L}} = \frac{1/R}{1/R + j\left(\omega C - \frac{1}{\omega L}\right)}$$

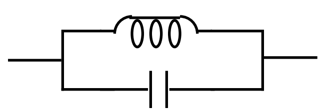
   b. Use your result in part (a) to obtain a general expression for the resonant frequency ω_p. Note that you don't have to take any derivates to find ω_p. Since the numerator is constant, you only have to find where the denominator is minimum.

   c. Verify that your result in part (b)

$$\omega_p = \frac{1}{\sqrt{LC}}$$

   gives the same value gives the same value for ω_p as your graph in Problem (5) for the corresponding values of L and C.

   d. Verify that the impedance of a parallel LC circuit as follows



   is infinite at resonance. **Memorize** this result.

   e. Given that the 3dB frequencies of a parallel RLC circuit are as follows

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \qquad \text{and} \qquad \omega_2 = +\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

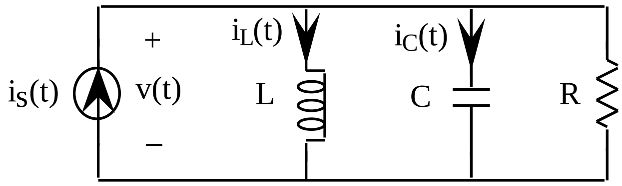
   find the **3dB Bandwidth** = $\omega_2 - \omega_1$.

   f. Verify that your result in part (e) gives the same value gives the same value for the 3dB Bandwidth as your graph in Problem (5) for the corresponding values of L and C.

   g. And finally - express the general equation in part (a) in **standard form** as follows

$$G(j\omega) = \frac{K(j\omega)}{(j\omega)^2 + a(j\omega) + b}$$

7. The objective of this problem is to find the relation between $i_L(t)$ and $i_C(t)$ at resonance in parallel RLC circuits as follows



a. Find $I_L(j\omega)$ and $I_C(j\omega)$ in terms of $V(j\omega)$
b. Make use of your result in part (a) to show that $i_L(t)$ and $i_C(t)$ are 180 degrees out of phase.
c. Make use of your result in part (b) to sketch $i_L(t)$ and $i_C(t)$ on the same graph at a frequency $\omega < \omega_p$
d. Make use of your result in part (b) to sketch $i_L(t)$ and $i_C(t)$ on the same graph at a frequency $\omega = \omega_p$
e. Make use of your result in part (b) to sketch $i_L(t)$ and $i_C(t)$ on the same graph at a frequency $\omega > \omega_p$
f. Make use of your results in parts (c)-(e) to describe the relationship between $i_L(t)$ and $i_C(t)$ as ω increases from $\omega < \omega_p$ to $\omega > \omega_p$