ECE 207 – FIRST ORDER RC CIRCUITS – INVESTIGATION 9
INTRODUCTION TO CAPACITORS

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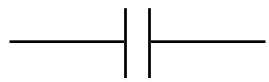
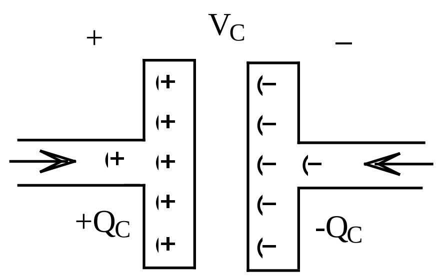
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

So far so good. We've investigated circuits containing resistors and controlled sources that can amplify and attenuate. But as useful as these circuits are, they can't separate one signal from another - like one radio or TV station from another. And they can't store information - they don't have memories. To do these and other related kinds of things we need capacitors and inductors. The objective of this investigation is to introduce linear capacitors - circuit elements that consist simply of two conductors - like two metal plates as follows



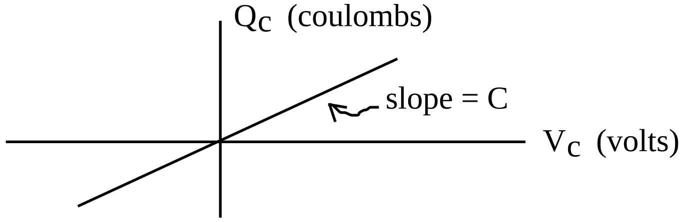
that are close but not touching.

1. As we know from our investigations on resistor circuits, the voltages across resistors are associated with currents flowing through them. But for capacitors, on the other hand, the voltages across them are as a result of the charges on their plates as follows



How would you expect the adding of more charge to the plates as indicated in the diagram affects the voltage V_C across the plates - would you expect V_C to increase, decrease or stay the same.

2. The **characteristic curves** of capacitors are graphs that show how Q_C varies as a function of V_C. **Linear capacitors**, in particular, have characteristic curves as follows



with Q_C proportional to V_C. We refer to C as the **capacitance** of the capacitor.
   a. Find C as a function of Q_C and V_C.

b. What will happen to V_C if Q_C doubles.
c. What are the units of C

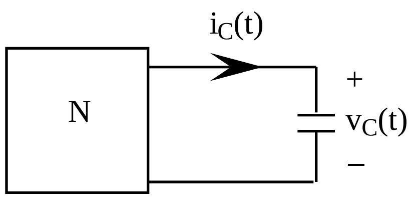
3. Given a linear capacitor of value C
   a. Find C in farads if $Q_C = 10^{-6}$ coul when $V_C = 2$ volts. Note that **farad** is shorthand for coul/volt with

$$1 \text{ farad} = \frac{1 \text{ coul}}{\text{volt}}$$

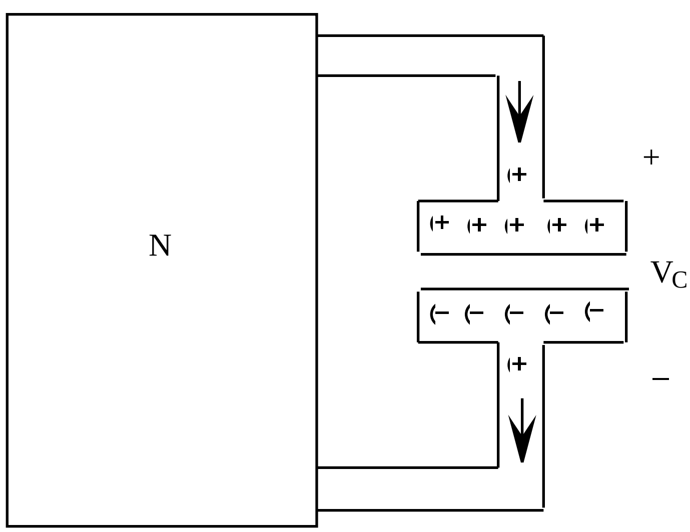
   Also note that $1\ \mu F = 10^{-6}$ F.
   b. What will be the charge on the plates when $V_C = 10$ volts.

4. Suppose we have two capacitors C_1 and C_2 with $C_1 > C_2$
   a. Sketch their characteristic curves on the same graph
   b. Which capacitor has the larger **capacity** to store charge - which capacitor, in other words, will have more charge on its plates for a given voltage across it plates.

5. The objective of this and the next several problems is to investigate the relationship between $i_C(t)$ and $v_C(t)$ for capacitors. Given the following circuit

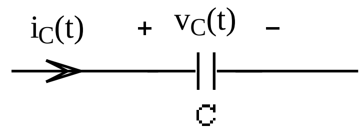


with epc flowing as follows when $i_C(t) > 0$



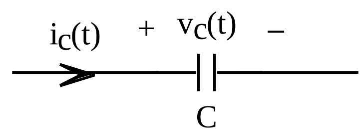
   a. What's happening to $v_C(t)$ in the above circuit with epc as indicated
   b. Redraw the above circuit showing the flow of the epc when $v_C(t) > 0$ and $i_C(t) < 0$
   c. Redraw the above circuit showing the flow of the epc when $v_C(t) < 0$ and $i_C(t) > 0$

6. Given a linear capacitor as follows



a. Explain in words why $i_C(t)$ is zero when $v_C(t)$ is constant
b. Is the amplitude of $i_C(t)$ large or small when $v_C(t)$ is changing quickly. How do you know.

7. Given a linear capacitor C as follows



with

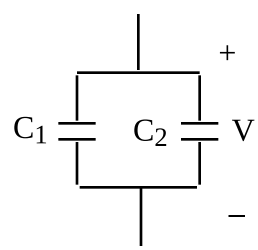
$$C = \frac{Q}{V} \qquad\qquad Q = CV$$

a. Make use of the general relationship

$$i(t) = \frac{dq(t)}{dt}$$

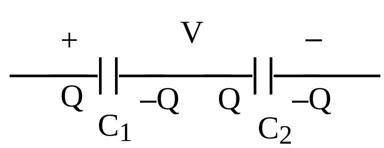
to come up with an equation for $i_C(t)$ in terms of $v_C(t)$ and the capacitance C. **Memorize** this relationship forever.
b. Make use of your result in part (a) to find $i_C(t)$ when $v_C(t) = V_o$ = constant
c. Find and sketch $i_C(t)$ when $C = 2\mu f$ and $v_C(t) = 5 \cos(2\pi 1000t + 1.2)$ volts

8. The objective of this and the next problem is to find equivalent capacitances. Given two capacitors C_1 and C_2 connected in parallel as follows



a. Express their equivalent capacitance $C_{eq} = Q_{eq}/V$ in terms of C_1 and C_2. Hint - make use of the fact that the total amount of charge being stored on the two capacitors is $Q_{eq} = Q_1 + Q_2$.
b. Put together 2 μf capacitors to form a 4μf capacitor.

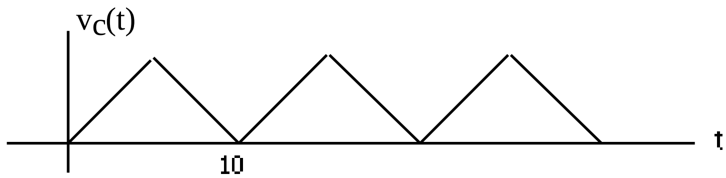
9. Given two capacitors connected in series as follows



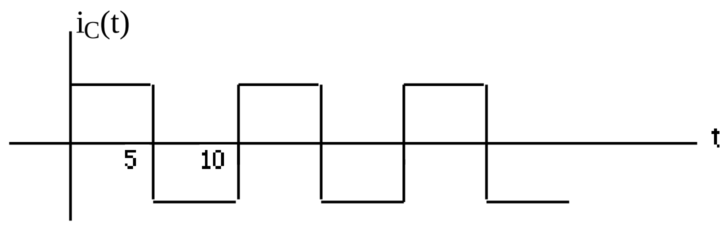
a. Express their equivalent capacitance $C_{eq} = Q_{eq}/V$ in terms of C_1 and C_2. Hint - make use of the fact that $Q_{eq} = Q_1 = Q_2$

b. Put together 2 µf capacitors to form a 5µf capacitor.

10. The objective of this and the next problem is to look at power and energy for capacitors. From ECE 109 we know that all the energy transferred to resistors is dissipated in the form of heat. Capacitors, on the other hand, store energy. Suppose, in particular, that we have a capacitor with voltage $v_C(t)$ as follows



and current

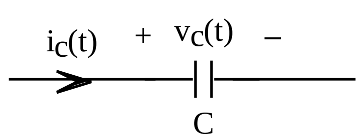


a. Sketch the power $p_C(t) = v_C(t)i_C(t)$
b. How is your curve in part (a) different from power curves for resistors
c. Then sketch the energy

$$E_C(t) = \int p_C(t)\,dt = \int v_C(t)i_C(t)\,dt$$

assuming $E_C(0) = 0$

11. Now suppose we have a current $i_C(t)$ flowing into an initially uncharged capacitor as follows



Then the energy stored in the electric field between the plates of the capacitor is given by

$$E_C(t) = \int p_C(t)\,dt = \int v_C(t)i_C(t)\,dt = \int v_C(t)\left(C\frac{dv_C(t)}{dt}\right)dt = \frac{1}{2}Cv_C^2(t)$$

Memorize this result. Then make use of it to

a. Calculate the energy stored in a 1µf capacitor when $v_C(t) = 5$ volts
b. Find and sketch the energy stored in a 1µf capacitor as a function of time when $v_C(t) = 5\cos(100t)$
c. By how much - by what factor - will the energy stored in a capacitor increase when the charge on the plates doubles