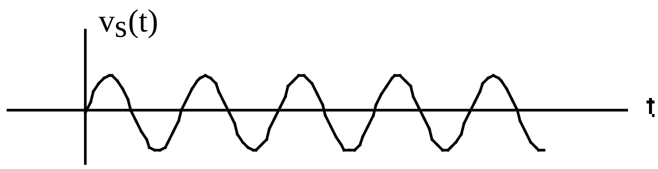
ECE 207 – CONTROLLED SOURCES – INVESTIGATION 4
INTRODUCTION TO CONTROLLED SOURCES

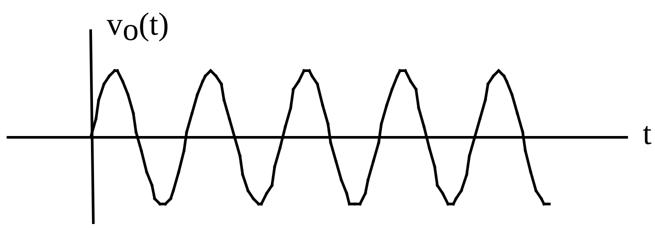
FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

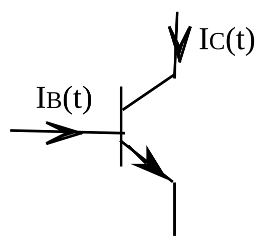
The objective of the first three investigations was to review the basics of resistor circuits - circuits that are very useful but also very limited. In particular, they can't amplify signals - they can't take small signals like the following



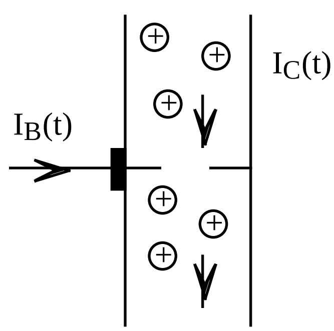
and produce larger replicas like the following



To build an amplifier we need a circuit element like the following transistor



where relatively small changes in the current $I_B(t)$ are able to control larger changes in the current $I_C(t)$ by "magically" controlling how much the flow of $I_C(t)$ is restricted as illustrated in the following diagram

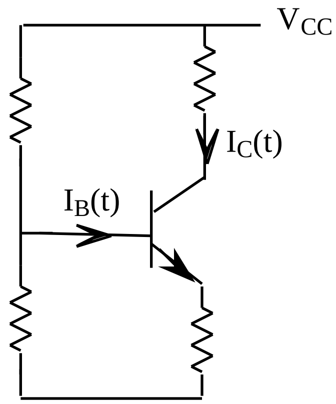


For such a transistor

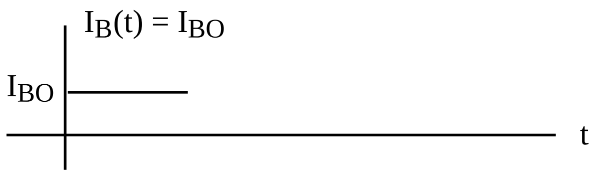
(a) The smaller $I_B(t)$ the smaller the "opening" and therefore the smaller $I_C(t)$

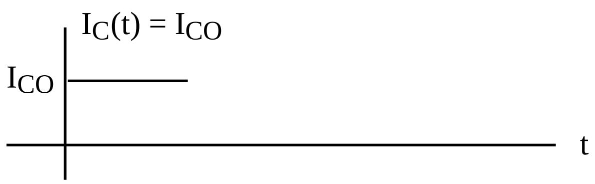
(b) The larger $I_B(t)$ the larger the "opening" and therefore the larger $I_C(t)$

To build an amplifier with such a transistor we first need to connect up resistors and a power supply as follows

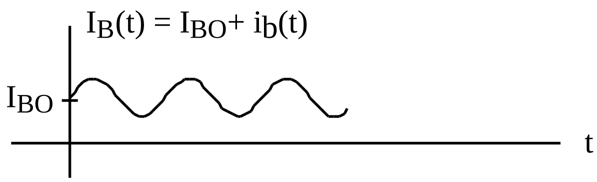


to produce constant values for $I_B(t)$ and $I_C(t)$ equal to I_{BO} and I_{CO} as follows

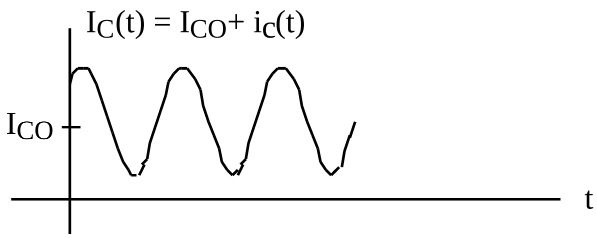




We refer to I_{BO} and I_{CO} as the transistor's **operating point**. We then connect to the circuit the small signal we would like to amplify - the signal that is going to do the controlling. Suppose we connect a small sinusoid. This will cause $I_B(t)$ to vary sinusoidally about its operating point by an amount $i_b(t)$ as follows



which in turn will cause $I_C(t)$ to follow suit as follows



with $i_c(t)$ **controlled** by $i_b(t)$ as follows

$$i_c(t) = k\, i_b(t)$$

with $k > 1$. And so we get the amplification we're after.

The objective of this investigation is to introduce controlled sources and show how they can be used to model the controlling of one signal by another. We will then make use of these controlled sources in subsequent investigations to analyze circuits that can amplify.

1. Suppose we have a transistor like that in the introduction above with

$$I_B(t) = I_{BO} + i_b(t) \quad \text{and} \quad I_C(t) = I_{CO} + i_c(t)$$

   where $I_{BO} = 30$ µa and $I_{CO} = 3$ ma are the operating points of I_B and I_C and $i_b(t)$ is the signal being amplified with

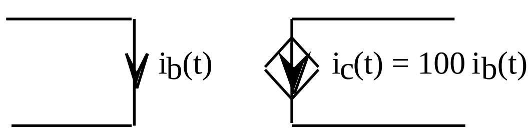
$$i_b(t) = 10 \cos(1000t)\ \mu a \quad \text{and} \quad i_c(t) = 100\, i_b(t)$$

   a. Sketch $I_B(t)$ and $I_C(t)$
   b. By how much is $i_b(t)$ being amplified – how much larger is $i_c(t)$ than is $i_b(t)$

2. For our transistor in Problem (1) we have $i_b(t)$ controlling $i_c(t)$ with

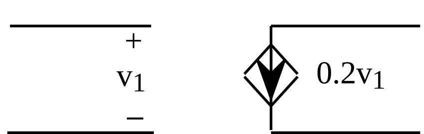
$$i_c(t) = 100\, i_b(t)$$

   where $i_c(t)$ is the changes in $I_C(t)$ about its operating point I_{CO} and $i_b(t)$ is the changes in $I_B(t)$ about its operating point I_{BO}. To **model** this control of one signal by another someplace else in the circuit we introduce a new class of circuit elements called **controlled (or dependent) sources** like the following



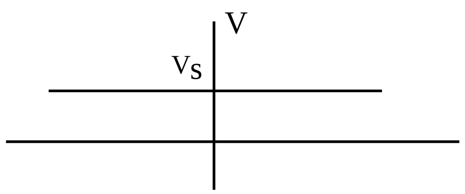
   which we refer to as a current controlled current source. Be sure to note that we use **diamonds** for controlled sources and include the controlling signal in our circuit drawing.

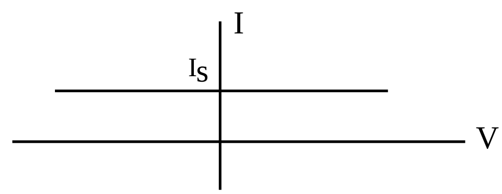
   What kind of controlled source is this



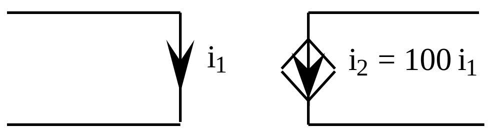
3. Draw a circuit diagram for a voltage controlled voltage source.

4. The objective of this problem is to show how to draw characteristic curves for controlled sources. From ECE 109 we know that ideal independent voltage sources with voltages $V = V_S$ have the same voltage no matter what the current I and ideal independent current sources with currents $I = I_S$ have the same current no matter what the voltage V as indicated in their characteristic curves as follows





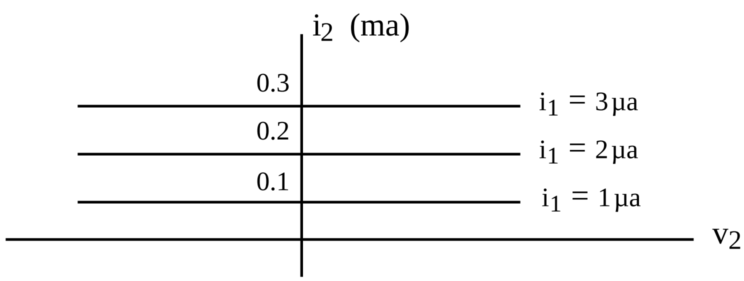
Controlled sources are characterized by the same kinds of curves - just more of them. Suppose, in particular, that we have a current controlled current source as follows



with

$$i_2 = 100\, i_1$$

Then we'll have a different horizontal line for each different value of i_1. To get a picture of what's going on we draw **families of curves**. For i_1 = 1µa, 2µa and 3µa, for example, we have the following family of curves

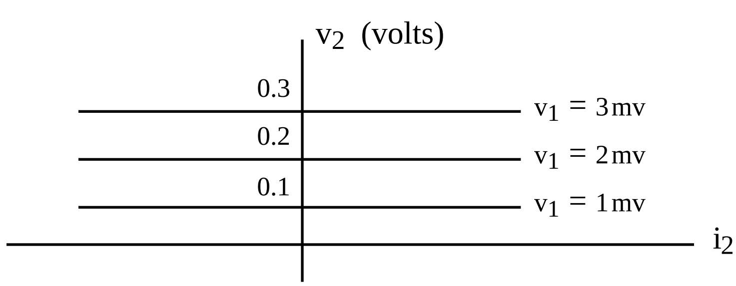


Draw the family of curves for the current controlled current source with

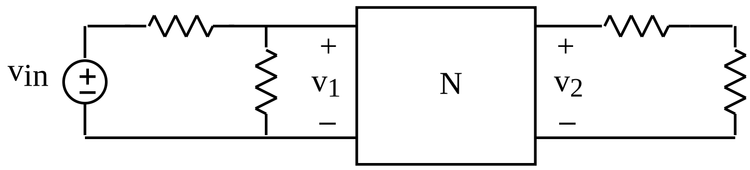
$$i_2 = 50 i_1$$

for i_1 = 1µa, 2µa and 3µa.

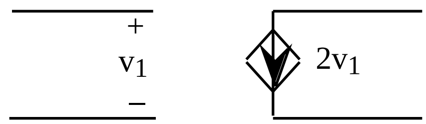
5. Draw the controlled source characterized by the following family of curves



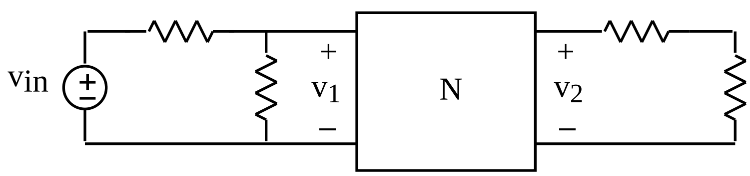
6. The objective of this and the next problem is to draw whole circuits with controlled sources. Redraw the following circuit



with N replaced by the following voltage controlled current source

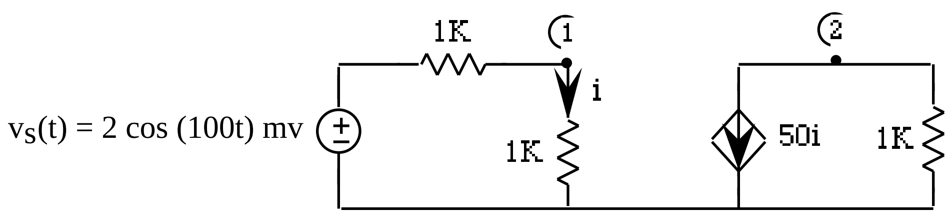


7. Redraw the following circuit



with N replaced by the controlled source characterized by $v_2 = 100v_1$. What kind of controlled source is this

8. The objective of this problem is to introduce how to analyze circuits with controlled circuits like those seen in electronic classes. If we simply write the node equations for such a circuit as follows



we obtain the following equations

<u>Node</u>	<u>Equation</u>
1	$\frac{v_1 - v_s}{1000} + \frac{v_1}{1000} = 0$
2	$50i + \frac{v_2}{1000} = 0$

which are great except that they have more unknowns than equations. The way we get around this problem, of course, is to simply express i in terms of the node voltages. We call this an **auxiliary equation**. Write the auxiliary equation for i and then solve for v_2.

9. Math Review Problem - One of the first things we do in algebra is find the equations of straight lines going through points with given slopes like that of the line going through the point (0, 1) with slope $m = 200$. Now your algebra teacher probably didn't tell you but you were really

solving your first differential equation as given by

$$x' = 200 \qquad x(0) = 1$$

What you were finding was the equation for the function x(t) with constant slope 200 and initial condition x(0) = 1. Now this was relatively easy but things get harder as the equations for the derivatives get more involved like in the following example

$$x' + 100x = 200 \qquad x(0) = 1$$

where x' depends not only on 200 but also x itself. Now from math we know that the complete solution to such a differential equation is the **sum** of a **homogeneous** part $x_h(t)$ and a **particular** part $x_p(t)$ as follows

$$x(t) = x_h(t) + x_p(t)$$

where $x_h(t)$ is the solution of the differential equation with the right hand side set to zero as follows

$$x'_h + 100x_h = 0$$

and $x_p(t)$ depends on the function on the right hand side of the differential equation. Note that in electrical engineering we usually refer to the homogeneous part as the **natural part $x_n(t)$** and the particular part as the **forced part $x_f(t)$** and so have

$$x(t) = x_n(t) + x_f(t)$$

Memorize this result forever.

The objective of this problem is to present a *cookbook* method for solving first order linear differential equations of the following general form

$$x' = ax + b$$

where b is a constant. The advantage of this method is that in a somewhat generalized form it can also be used to solve first order coupled differential equations as we will see.

a. The first step in finding the natural response $x_n(t)$ of our differential equation

$$x' = ax + b$$

is to write and solve the following corresponding equation for the variable s

$$s - a = 0$$

where a is the coefficient of x. The result, of course, is simply

$$s = a$$

The natural response is then, as can be shown in math, the exponential

$$x_n(t) = Ke^{st} = Ke^{at}$$

where K depends on the initial condition of the complete response as shown in part (c) below. Make use of this method to find the natural response of the following first order differential equation

$$x' + 500x = 2000 \qquad x(0) = -2$$

as a function of K. Note that we refer to the value of s as the **natural frequency** of the circuit. **Memorize** this definition. It's a very common term.

b. To find the forced part of the solution $x_f(t)$ - the part that satisfies

$$x'_f + 100x_f = 200$$

we make use of the fact that we "know" - from math and from experience in the lab - that the forced response of a first order linear differential equation to a **constant input** (the 200) is also a constant as follows

$$v_f(t) = A$$

To find A we simply substitute $v_f(t) = A$ into the differential equation as follows

$$\frac{d}{dt}A + 100A = 200$$

which gives us

$$0 + 100A = 200$$

and so $A = 2$. Therefore for this example

$$x_f(t) = A = 2$$

Use this scheme to find the forced response $x_f(t)$ of

$$x' + 500x = 2000 \qquad x(0) = -2$$

c. Once we have the natural and forced parts of a solution we can add them together and apply the initial conditions to get the complete solution. For our running example we have

$$x(t) = x_n(t) + x_f(t) = Ke^{-100t} + 2$$

and so plugging in the initial conditions we have $x(0) = 1 = K + 2$ $\quad K = -1$ and so

$$x(t) = x_n(t) + x_f(t) = -e^{-100t} + 2$$

Now find and plot the complete solution of the differential equation

$$x' + 500x = 2000 \qquad x(0) = -2$$

for $t > 0$