ECE 207 – FIRST RC ORDER CIRCUITS – INVESTIGATION 24
SINUSOIDAL STEADY STATE RESPONSES OF RC CIRCUITS

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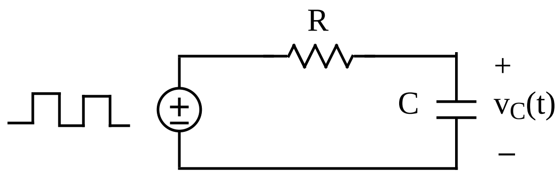
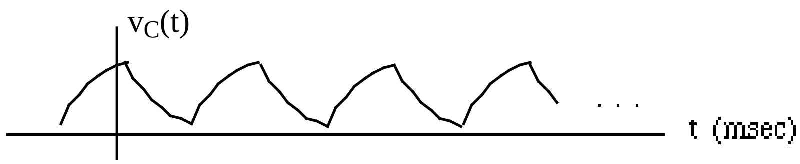
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From our previous investigation on RC circuits with pulse train inputs as follows



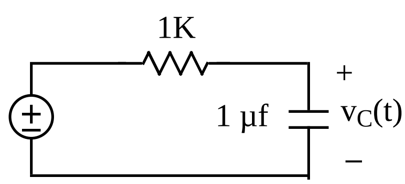
we know that the steady state response after the transient has decayed has the same frequency as the input but in general only "resembles" a pulse train as follows



The objective of this investigation is to see what happens when the input to our RC circuit is a sinuosid. As it turns out sinusoids are "magic" signals. Not only are the steady responses periodic with the same frequency as the input but they're also themselves sinusoids. The analysis problem then reduces to that of finding the steady state amplitude and phase.

1. We begin with a first order RC circuit with a sinusoidal input as follows

$v_S(t) = 5 \cos(2\pi 1000t)$



$v_C(0) = 1$ volt

   a. Sketch $v_S(t)$. What is its period
   b. Sketch a drawing to illustrate how the equivalent positive charges are flowing through the circuit when $v_S(t) > v_C(t)$. Then do the same for when $v_S(t) < v_C(t)$. Describe what's going on at the plates of the capacitor. Be sure to make clear that the equivalent positive charge does **not** flow through the capacitor.
   c. Find the time constant τ and then make use of it to sketch the natural response $v_n(t)$
   d. How many periods of the sinusoid will it take for the circuit to reach steady state
   e. Write the differential equation for $v_C(t)$
   f. Solve your differential equation in part (e) for the sinusoidal steady state response of $v_C(t)$ - the forced response of $v_C(t)$. Make use of the fact that $v_f(t)$ is a sinusoid of the same frequency as $v_S(t)$ as follows

$$v_f(t) = B \cos(2\pi 1000t + \varphi)$$

a result we not only observe in the lab but can also prove in math. So all we need to do is solve for B and φ. The way we do this is simply substitute $v_f(t) = B\cos(2\pi1000t + \varphi)$ into the circuit's differential equation

$$v_f' + 1000v_f = 1000v_s(t) = 5000\cos(2\pi1000t)$$

as follows

$$\frac{d(B\cos(2\pi1000t + \phi))}{dt} + 1000B\cos(2\pi1000t + \phi) = 5000\cos(2\pi1000t)$$

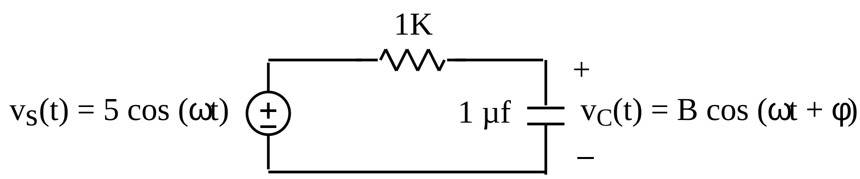
and then make use of the trig identity

$$C\cos(x + \theta) = D\cos(x) + E\sin(x) = \sqrt{D^2 + E^2}\cos\left(x + \tan^{-1}\left(\frac{-E}{D}\right)\right)$$

to solve for B and φ.

g. Now make use of your results in parts (c) and (f) to find and sketch the complete response of $v_C(t)$

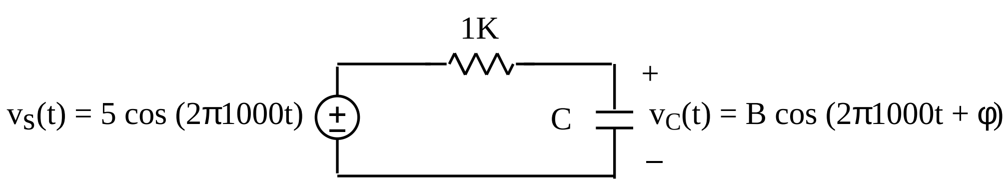
2. In Problem (1) we found the sinusoidal steady state response of a circuit for a given sinusoidal input. The objective of this problem is to see what happens to the sinusoidal steady state response as we vary the frequency. Given our first order RC circuit with steady state response $v_C(t) = 5\cos(\omega t + \varphi)$ as follows



a. Find and sketch B as a function of ω

b. Explain in words why B decreases as ω increases. **Memorize** this result.

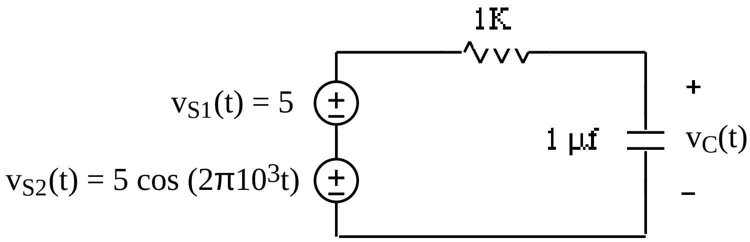
3. In Problem (2) we saw what happened to the sinusoidal steady state response of our circuit as we varied the frequency. The objective of this problem is to see what happens to the sinusoidal steady state response as we vary the value of the capacitor C. Given our first order RC circuit with sinusoidal steady response $v_C(t) = B\cos(2\pi1000t + \varphi)$ as follows



Explain in words why the amplitude of $v_C(t)$ decreases as C increases

4. How do the initial conditions of an RC circuit affect the amplitude and phase of its sinusoidal steady response.

5. Make use of your memorized result that steady state (forced) responses of linear circuits satisfy superposition - just like the resistor circuits of ECE 109 do - to find and sketch the steady state response of the following circuit. Be sure to make use of your calculations in Problem (1)


1K
$v_{S1}(t) = 5$
$v_{S2}(t) = 5 \cos(2\pi 10^3 t)$
1 µf
+
$v_C(t)$
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