ECE 207 - 2ND ORDER CIRCUITS - INVESTIGATION 23
SECOND ORDER RLC CIRCUITS - PART III

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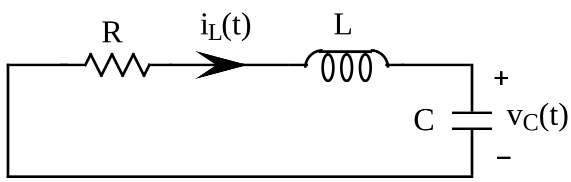
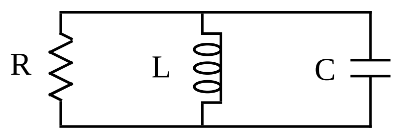
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

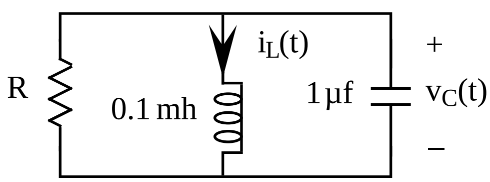
From the last two investigations we know that second order series RLC circuits as follows



can be overdamped, underdamped or critically damped depending on the relative sizes of R, L and C. We saw, in particular, that a series RLC circuit with a relatively small R is underdamped with natural frequencies that are complex conjugates. And a series RLC circuit with a relatively large R is overdamped with natural frequencies that are real and different. The main objective of this investigation is to calculate the natural responses of other second order RLC circuits such as the following parallel circuit



1. Given the following second order parallel RLC circuit

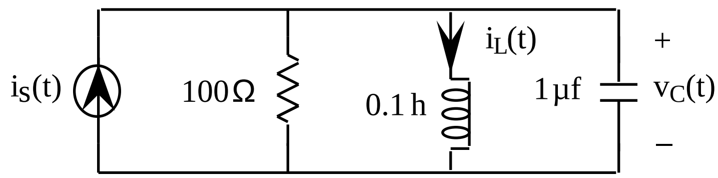


   a. Show that the first order coupled differential equations for this circuits are as follows

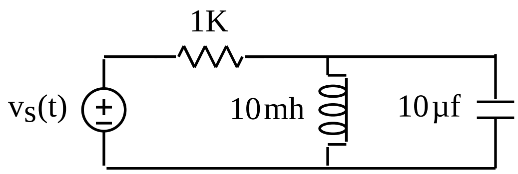
$$i_L' = 10^4 v_C \qquad v_C' = -10^6 i_L - \frac{10^6}{R} v_C$$

   b. Find a value of R that makes the circuit underdamped
   c. Find a value of R that makes the circuit overdamped
   d. From parts (b) and (c) we see that in contrast to series RLC circuits, parallel RLC circuits are underdamped when R is large and overdamped when R is small. Explain in words why this is so.

2. Find and sketch the step responses of $i_L(t)$ and $v_C(t)$ in the following second order parallel RLC circuit

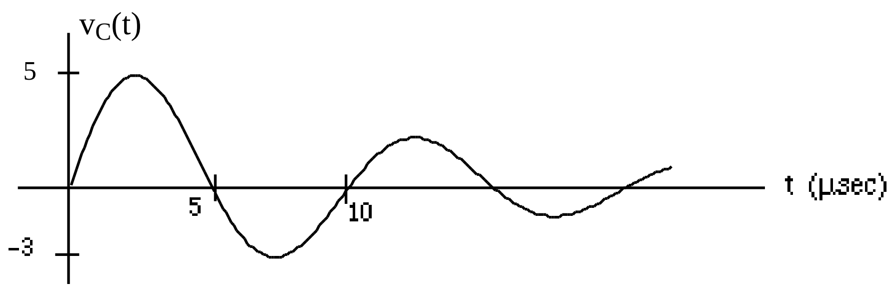


3. Given the following circuit



   a. Write and put in matrix form the first order coupled differential equations
   b. Is this circuit overdamped, underdamped or critically damped

4. Find the natural frequencies of a second order RLC circuit with step response as follows



5. Math Review - Sketch each of the following signals
   a. v(t) = 5 cos (2π1000t + 1.2)
   b. v(t) = 5 cos (2π1000t - 1.2)

6. Math Review - Find A and θ in the following equation if for all time t

$$A\cos(1000t + \theta) = 3\cos(1000t + 1.2)$$

7. Math Review - Find each of the following angles in radians in the range $-\pi \le \theta \le \pi$. Be careful - in each case draw a picture showing the quadrant the angle is in. You should get one angle in each quadrant

   a. $a = \tan^{-1}\left(\frac{2}{3}\right)$ c. $c = \tan^{-1}\left(\frac{2}{-3}\right)$

   b. $b = \tan^{-1}\left(\frac{-2}{3}\right)$ d. $d = \tan^{-1}\left(\frac{-2}{-3}\right)$

8. Math Review - Make use of the trig identity

$$A\cos(x+\theta) = B\cos(x) + C\sin(x) = \sqrt{B^2 + C^2}\,\cos\left(x + \tan^{-1}\left(\frac{-C}{B}\right)\right)$$

   to find A and θ in the following equation

$$A\cos(1000t + \theta) = 3\cos(1000t + 0.5) + 4\sin(1000t + 0.5)$$