# ECE 207 - 2ND ORDER CIRCUITS - INVESTIGATION 22
## SECOND ORDER RLC CIRCUITS - PART II

**FALL 2000**                                                                **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We saw in the last investigation that if the resistor in a series RLC as follows
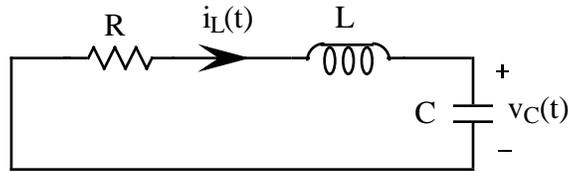


is relatively small then the natural frequencies are complex conjugates and the natural response underdamped as follows

$$Ke^{-at}\cos(bt+\theta)$$

The objective of this investigation is to see how increasing the value of the series resistor affects the circuit's response.

1. Given an underdamped RLC series circuit as follows with $i_L(0) > 0$, $v_C(0) > 0$
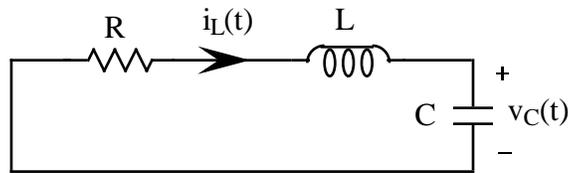


   a. What would you expect happens to $v_C(t)$ and $i_L(t)$ as R increases from a small to a large value. Draw a series of sketches to illustrate your answer.
   b. Explain how a mechanical system consisting of a spring connected to a mass that is in a container of oil as follows
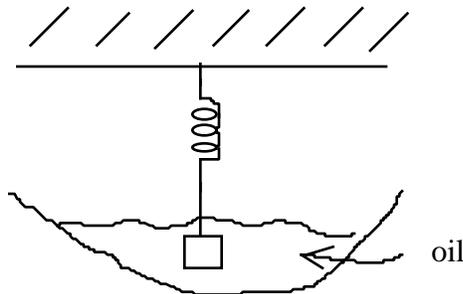


   is analogous to our series RLC circuit. How, in particular, is the viscosity of the oil related to the size of the resistor.

2. The objective of this problem is to analyze the 2nd order series RLC circuit from the last investigation but with a larger resistor as follows

2K  $i_L(t)$  0.1 h

1 µf  $v_C(t)$
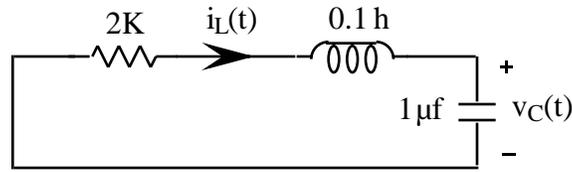
a. Find and put in matrix form the coupled first order differential equations.
b. Make use of your result in part (a) to find the natural frequencies of the circuit.
c. When the natural frequencies of a circuit are real and different as follows

$$s_1 = a \qquad s_2 = b$$

like they are for this circuit then the natural responses of $i_L(t)$ and $v_C(t)$ are a sum of complex exponentials as follows

$$K_1 e^{at} + K_2 e^{bt}$$

And we say the circuit is **overdamped**. **Memorize** the equation for the natural response when the natural frequencies are real and different and the circuit is overdamped. The values of $K_1$ and $K_2$ for $i_L(t)$ and $v_C(t)$ depend on the initial conditions of their complete responses. Write out the equation for $v_C(t)$ as a function of $K_1$ and $K_2$

e. Find $v'_C(0)$ and $i'_L(0)$ if the initial conditions of the circuit are

$$v_C(0) = 2 \text{ volts} \qquad i_L(0) = -5 \text{ ma}$$

f. Make use of your results in parts (d) and (e) to calculate $i_L(t)$ and $v_C(t)$.
g. Now make use of your results in part (f) to sketch $i_L(t)$ and $v_C(t)$.

3. From the results in Problems (1) and (2) we know that as the value of R increases in a series RLC the natural response goes from being underdamped with natural response of the form

$$Ke^{at} \cos(bt + \theta)$$

to being **overdamped** with natural response equal to a sum of decaying exponentials like the following

$$K_1 e^{at} + K_2 e^{bt}$$

a. How do the natural frequencies of overdamped circuits differ from those of underdamped circuits. **Memorize** this relation
b. Describe in words how the graph of an underdamped response looks different from that of an overdamped circuit

4. The objective of this problem is to compare the step responses of overdamped and underdamped series RLC circuits



R  $i_L(t)$  L

$v_S(t)$  C  $v_C(t)$
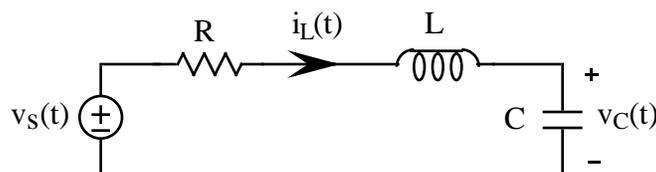
a. Sketch the step responses of $i_L(t)$ and $v_C(t)$ if the circuit is underdamped.

b. How long will it take the following underdamped step response to reach steady state
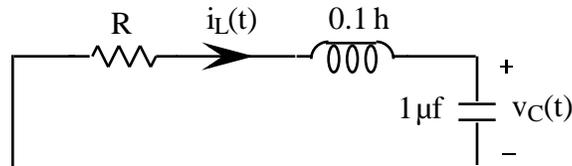
$$v_C(t) = 5 + 2e^{-10^3 t} \cos(2 \cdot 10^4 t + 1.2)$$

c. Sketch the step responses of $i_L(t)$ and $v_C(t)$ if the circuit is overdamped.

d. How long will it take the following overdamped step response to reach steady state

$$v_C(t) = 5 + 2e^{-10^3 t} - 1.5e^{-10^4 t}$$

5. When the natural frequencies of a circuit like the following 2nd order series RLC circuit



are real and equal as follows

$$s_1 = s_2 = a$$

then the natural response of the circuit is of the form

$$K_1 e^{at} + K_2 t e^{at}$$

and we say the circuit is **critically damped**. **Memorize** what it means for a circuit to be critically damped and then find the value of R that makes this circuit critically damped. A circuit that is critically damped is just on the **borderline** between underdamped and overdamped. For R any smaller in our **series** circuit, it will be underdamped and for any larger overdamped.

6. Math Review - Make use of the trig identity

$$A \cos(x + \theta) = B \cos(x) + C \sin(x) = \sqrt{B^2 + C^2} \, \cos\left[ x + \tan^{-1} \frac{-C}{B} \right]$$

to find A and $\theta$ in the following equation

$$A \cos (1000t + \theta) = 3 \cos (1000t + 0.5) + 4 \sin (1000t + 0.5)$$