ECE 207 - 2ND ORDER CIRCUITS - INVESTIGATION 21
SECOND ORDER RLC CIRCUITS - PART I

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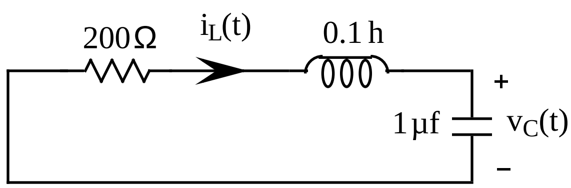
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to see what happens when we add a relatively small series resistor to an LC tank circuit.

1. Suppose we insert a relatively small series resistor R to an LC tank circuit as follows



a. Make sketches of what you expect $i_L(t)$ and $v_C(t)$ to look like. Why do you think your graphs look the way they do.
b. Verify that the coupled first order equations of this circuit are equal to

$$v_C' = 10^6 i_L \qquad i_L' = -10 v_C - 2000 i_L$$

c. Put your coupled differential equations from part (b) in matrix form and then find the natural frequencies.
d. When the natural frequencies of a circuit are complex conjugates of the form

$$s_1 = a + jb \qquad s_2 = s^*_1 = a - jb$$

like they are for this circuit then the natural responses of $i_L(t)$ and $v_C(t)$ are damped sinusoids of the form

$$Ke^{at}\cos(bt + \theta)$$

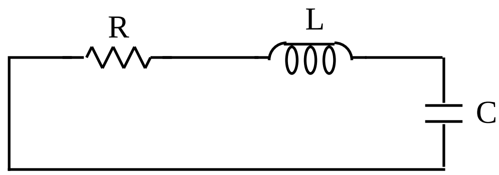
And we say the circuit is **underdamped**. **Memorize** the equation for the natural response when the natural frequencies are complex conjugates and the circuit is underdamped. The values of K and θ for $i_L(t)$ and $v_C(t)$ depend on the initial conditions of their complete responses. Write out the equation for $v_C(t)$ as a function of K and θ

e. Use the coupled first order differential equations in part (b) to find $v'_C(0)$ and $i'_L(0)$ if the initial conditions of the circuit are

$$v_C(0) = 2 \text{ volts} \qquad i_L(0) = -5 \text{ ma}$$

f. Now make use of your results in parts (d) and (e) to sketch $i_L(t)$ and $v_C(t)$.
g. Do your results in part (f) agree with your original conjectures in part (a). If not explain your mistake in part (a).
h. How did adding the resistor to the LC tank affect its frequency.

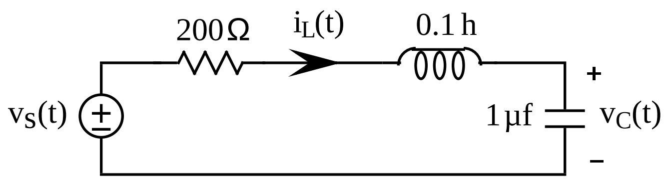
2. Can the total energy E in an RLC circuit as follows



sometimes be increasing as follows

Explain in words how you know.

3. The objective of this problem is to find the unit step response of the series RLC circuit in Problem (1) as follows



a. First find $i_L(0+)$, $v_C(0+)$, $i'_L(0+)$, $v'_C(0+)$ and $v_L(0+)$ of the unit step response. Remember that for the unit step response
   (1) $v_S(t) = u(t)$ = unit step
   (2) All initial conditions are equal to zero with $i_L(0-) = 0$ and $v_C(0-) = 0$

b. Find the steady state values of $i_L(t)$ and $v_C(t)$. Hint: replace the inductors by shorts and the capacitors by opens as we have done before

c. Now make use of the fact that the circuit is underdamped together with your results in parts (a) and (b) to sketch $i_L(t)$ and $v_C(t)$

4. Given the following response of an underdamped circuit

$$v_C(t) = 5 + 5e^{-10^4 t}\cos(2\pi 10^4 t + 0.7)$$

a. What is the steady state value of $v_C(t)$
b. How long will it take the transient part of the response to decay to zero
c. Sketch $v_C(t)$. Be sure to calculate and make use of $v_C(0)$ and $v'_C(0)$