ECE 207 - 2ND ORDER CIRCUITS - INVESTIGATION 20
SECOND ORDER LC CIRCUITS

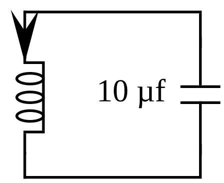
FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

As we have seen in our previous investigations, the voltages and currents of first order RC and RL circuits like the following

decay to zero as the resistors dissipate the energy. The objective of this investigation is to see what happens in LC circuits that contain no resistors.

1. We begin by seeing what happens in the following second order LC circuit



   a. Verify that the first order coupled differential equations for this circuit are

$$v_C' = -10^5 i_L \qquad i_L' = 10^3 v_C$$

   b. Make use of your results in part (a) to find $i'_L(0)$ and $v'_C(0)$ if the circuit's initial conditions are

$$i_L(0) = 5 \text{ ma} \qquad \text{and} \qquad v_C(0) = 0 \text{ volts}$$

   c. Now put your equations from part (a) in matrix form and find the natural frequencies.
   d. When the natural frequencies of a circuit are purely imaginary with

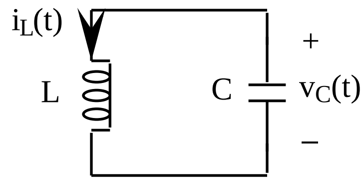
$$s_1 = +j\omega_o \qquad s_2 = -j\omega_o$$

   as they are for our LC circuit then the natural responses of $v_C(t)$ and $i_L(t)$ are pure sinsusoids of the form

$$K \cos(\omega_o t + \theta)$$

   where K and θ depend on the initial conditions of the complete response - which for circuits like ours without sources is simply the natural response. **Memorize** this result. And then make use of it and your results in part (b) to find and sketch $v_C(t)$ and $i_L(t)$
   e. Explain in words why $v_C(t)$ and $i_L(t)$ don't decay to zero in this LC circuit like they do in RC and RL circuits like those in the introduction.

2. The objective of this problem is to generalize on the results of Problem (1) with the following circuit



a. Verify that the coupled first order differential equations for this circuit are

$$v'_C = -\frac{1}{C}\, i_L \qquad \text{and} \qquad i'_L = \frac{1}{L}\, v_C$$

b. Make use of the equations from part (a) to verify that the natural frequencies of the circuit are purely imaginary with values

$$s_1 = j\frac{1}{\sqrt{LC}} \qquad s_2 = -j\frac{1}{\sqrt{LC}}$$

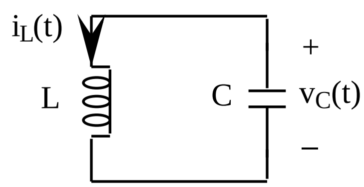
and so the natural response is a pure sinusoid with frequency

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Memorize this result.

c. How does increasing C and/or L affect the frequency of an LC circuit.

3. The objective of this problem is to show that as $v_C(t)$ and $i_L(t)$ oscillate sinusoidally the capacitor and inductor in an LC circuit as follows



are transferring their energies back and forth.

a. Show that when the magnitude of $v_C(t)$ is increasing then the inductor is transferring energy to the capacitor. Hint - show that when the magnitude of $v_C(t)$ is increasing then the magnitude of $i_L(t)$ is decreasing.
b. Show that when the magnitude of $v_C(t)$ is decreasing then the inductor is receiving energy from the capacitor.
c. Show on a graph of $v_C(t)$ where the capacitor is receiving energy from the inductor and where it's transferring energy to the inductor.
d. Show on a graph of $i_L(t)$ where the inductor is receiving energy from the capacitor and where it's transferring energy to the capacitor.
e. Why do you think we refer to LC circuits like the one in this investigation as an LC tank
f. Verify that no energy is being lost in the ideal circuit of Problem (1) - that its total energy $E(t) = E_C(t) + E_L(t)$ is always constant.