ECE 207 – FIRST ORDER RL CIRCUITS – INVESTIGATION 19
INTRODUCTION TO FIRST ORDER RL CIRCUITS

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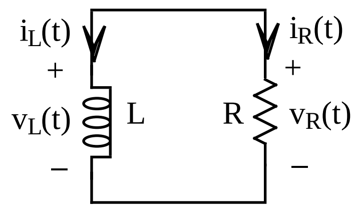
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The main objective of this investigation is to calculate the responses of first order RL circuits with constant inputs, step inputs and switches.

1. Let us begin with the following first order RL circuit



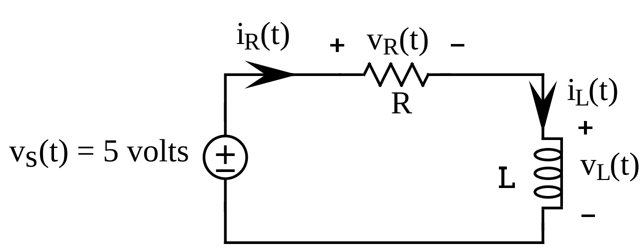
with initial condition $i_L(0) = 2$ ma

a. What is $i_R(0)$
b. What will happen to the inductor's energy and therefore its current as time passes. Explain in words how you know.
c. Sketch graphs of what you expect for $i_L(t)$, $v_R(t)$ and $v_L(t)$. Describe your curves
d. Make use of KVL and the following equation for the inductor

$$v_L(t) = L\frac{di_L(t)}{dt}$$

to obtain the differential equation for $i_L(t)$ for R = 1K and L = 1 mh
e. Solve your differential equation in part (d) for $i_L(t)$. Sketch your result.
f. Now make use of your result in part (e) to find and sketch $v_L(t) = v_R(t)$.
g. Are your results in parts (e) and (f) consistent with your original conjectures in part (c). If not, explain what in fact is going on

2. Now suppose we have a first order RL circuit with a constant input as follows

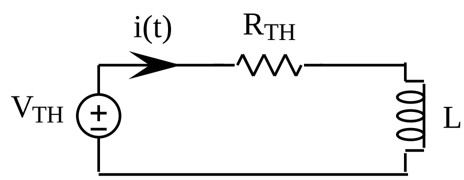


and initial condition $i_L(0) = 0$

a. What are $i_R(0)$ and $v_L(0)$
b. Make use of your result in part (a) for $v_L(0)$ to show that the inductor's current $i_L(t)$ and therefore its energy are increasing with time

c. Sketch graphs of what you expect for $i_L(t)$, $v_R(t)$ and $v_L(t)$. Describe your curves
d. Make use of KVL around the loop to obtain the differential equation for $i_L(t)$ when R = 1K and L = 1 mh
e. Solve your differential equation in part (d) for $i_L(t)$. Sketch the natural, forced and complete responses.
f. Find and sketch $v_R(t)$ and $v_L(t)$. What is the steady state value of $v_L(t)$
g. Are your results in parts (e) and (f) consistent with your original conjectures in part (c). If not, explain what in fact is going on

3. The objective of this problem is to find a general expression for the time constant τ of a first order RL circuit of the following general form

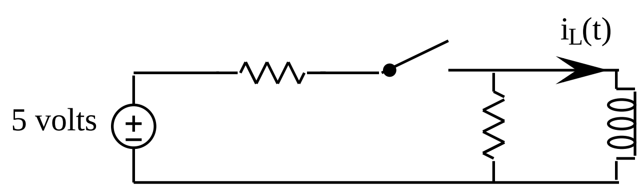


a. First write the differential equation for i(t)
b. Then make use of your result in part (a) to show that the circuit's time constant is

$$\tau = \frac{L}{R_{TH}}$$

Memorize this result.
e. How long would it take the circuit of Problem (2) to reach steady state if R_{TH} = 2K and L = 5 mh

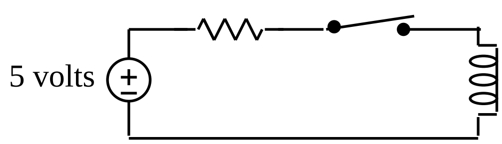
4. The objective of this and the next problem is to develop a basic property of inductor circuits with switches. The first observation is that if a switch opens or closes at time t = 0 in a circuit like the following



then the inductor current will not change instantaneously. In particular

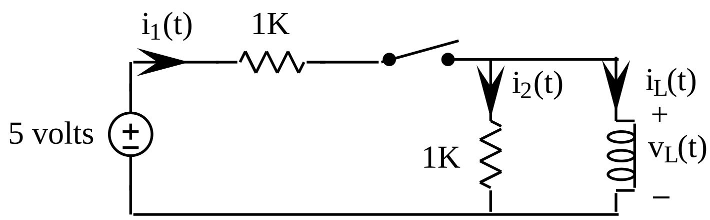
$$i_L(0-) = i_L(0+)$$

where $i_L(0-)$ is the current through the inductor just before the switch opens or closes and $i_L(0+)$ is the current just after the switch opens or closes. The reason for this result is that it takes time for an inductor's magnetic field to change - as we've seen in our calculations. The only "exception" is, of course, in circuits like the following



which will generate sparks if you open the switch.

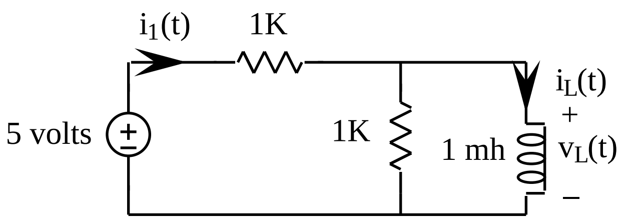
Given the following circuit



with the switch opening at time $t = 0$ and $i_L(0-) = 2$ ma

a. Find $i_1(0-)$, $i_2(0-)$ and $v_L(0-)$
b. Find $i_L(0+)$, $i_1(0+)$, $i_2(0+)$ and $v_L(0+)$

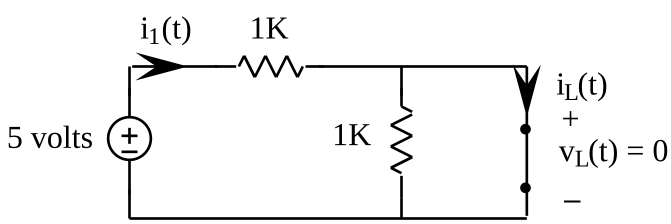
5. The objective of this problem is to see what's going in RL circuits with **constant inputs** like the following



$i_L(0) = 0$

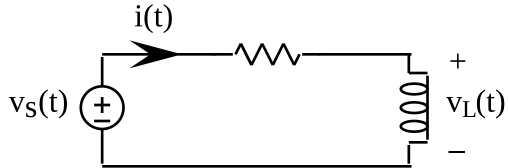
after the circuit reaches **steady state**

a. Find and sketch $i_L(t)$ and $v_L(t)$
b. What are $i_L(t)$, $v_L(t)$ and $i_1(t)$ when the circuit is in steady state
c. From parts (a) and (b) we see that once the circuit reaches steady state $v_L(t) = 0$. What this means is that once an RL circuit with a **constant input** is in the steady state then we can analyze the circuit with the inductors replaced by short circuits as follows



Memorize this result. Then make use of it to calculate the steady state values of $i_1(t)$ and $i_L(t)$. Make sure you get the same results as you got in part (b).

6. Make use of your results in Problems (4) and (5) to sketch the step responses i(t) and $v_L(t)$ in the following first order RL circuit



7. Sketch $v(t) = Ke^{-1000t}\cos(2\pi 1000t + \theta)$ if $v(0) > 0$ and $v'(0) > 0$