# ECE 207 - FIRST ORDER RL CIRCUITS - INVESTIGATION 18
# INTRODUCTION TO INDUCTORS

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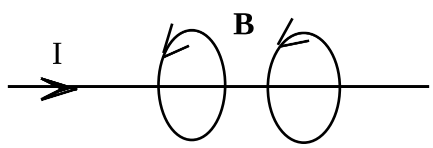
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

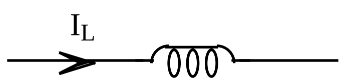
The goal of the last series of investigations was to explore the basic properties of circuits containing linear capacitors. The objective of this and the next investigation is to make use of the following basic properties of currents and their magnetic fields to introduce linear inductors and the circuits containing them -

(1) Currents I flowing through wires generate "magical" magnetic fields **B** (in teslas) that encircle the wires as follows




As you would expect, the larger I the larger **B**. We know these fields "exist" because charges moving through them feel the forces they exert

(2) Inductors as follows




are simply coils of wire. We wrap wire in coils in order to concentrate the magnetic field in as small a volume as possible

(3) The magnetic flux λ through a surface is - loosely speaking - the area A of the surface times the magnetic field B going through the surface. The magnetic flux λ of an inductor is the sum of the fluxes through each of its loops. The units of flux is webers = tesla·$m^2$

(4) Energy is stored in the magnetic fields of inductors just as energy is stored in the electric fields of capacitors. The larger the current and therefore the larger the magnetic flux λ, the more the energy that is being stored

(5) It takes work to change the magnetic flux of an inductor - to change the amount of energy it's storing. By Faraday's Law we have that the voltage v(t) across an inductor - the amount of energy transferred by each coulomb of charge passing through it - is related to the rate the magnetic flux is changing by

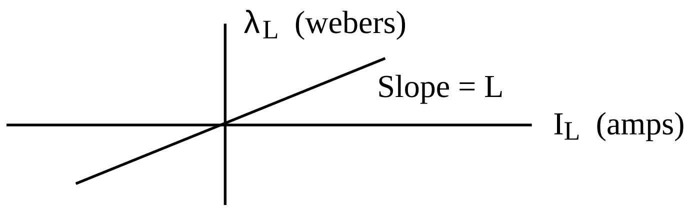
$$v(t) = \frac{d\lambda(t)}{dt}$$

From Faraday's Law we have that a changing magnetic flux will generate a voltage across an inductor and conversely a voltage across an inductor will cause its magnetic flux and therefore its current to change

The objective of this investigation is to make use of these results to develop the basic properties of linear inductors.

1. How would you expect an increase in the magnitude of the current through an inductor to affect the magnitude of its magnetic flux $\lambda$ - would you expect $\lambda$ to increase, decrease or stay the same. How do you know.

2. The **characteristic curves** of inductors are graphs that show how $\lambda_L$ varies as a function of $I_L$. **Linear inductors**, in particular, have characteristic curves as follows




with $\lambda_L$ proportional to $I_L$. We refer to L as the **inductance** of the inductor.

   a. Find L as a function of $\lambda_L$ and $I_L$
   b. What will happen to $I_L$ if $\lambda_L$ doubles
   c. What are the units of L

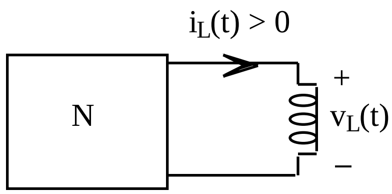
3. Given a linear inductor of value L
   a. Find L in henrys if $\lambda = 10^{-6}$ webers when I = 2 ma. Note that henry is shorthand for webers/amp with

$$1 \text{ henry} = \frac{1 \text{ weber}}{\text{amp}}$$

   b. What will be the flux $\lambda$ when I = 5 ma

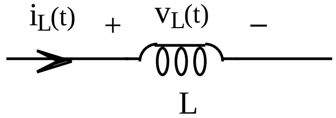
4. Suppose we have two inductors $L_1$ and $L_2$ with inductances $L_1 > L_2$.
   a. Sketch their characteristic curves on the same graph
   b. Which inductor would you expect has the most turns. How do you know

5. The objective of this and the next several problems is to investigate the relationship between $v_L(t)$ and $i_L(t)$ for inductors. Suppose we have a linear inductor connected to a circuit N as follows



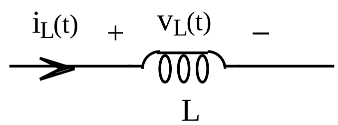

   a. Will $v_L(t) > 0$ cause $\lambda_L(t)$ to increase or decrease. How do you know
   b. Repeat part (a) for $v_L(t) < 0$

6. Given a linear inductor as follows



a. What is $v_L(t)$ when $i_L(t)$ is constant. How do you know
b. Is the magnitude of $v_L(t)$ large or small when $i_L(t)$ is changing quickly. How do you know.

7. Given a linear inductor as follows




with

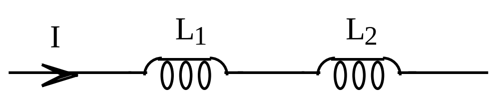
$$L = \frac{\lambda}{I} \qquad\qquad \lambda = LI$$

a. Make use of Faraday's Law

$$v_L(t) = \frac{d\lambda(t)}{dt}$$

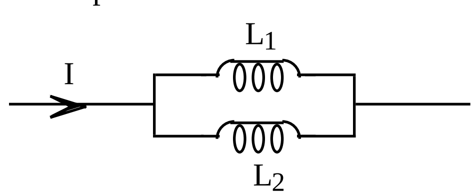
to obtain an equation for $v_L(t)$ as a function of $i_L(t)$ and the inductance L. **Memorize** this result forever.
b. Make use of your result in part (a) to find $v_L(t)$ when $i_L(t) = I_o$ = constant
c. Find and sketch $v_L(t)$ when L = 0.2 mh and $i_L(t) = 5 \cos(2\pi 10^6 t - 1.2)$ ma

8. The objective of this and the next problem is to find equivalent inductors. Given two inductors $L_1$ and $L_2$ connected in series as follows



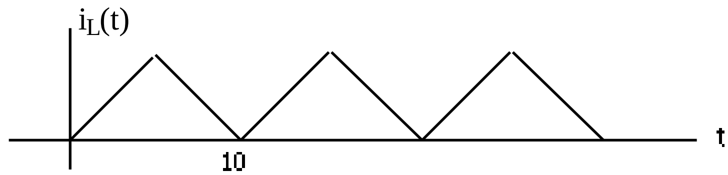

a. Express the equivalent inductance $L_{eq} = \lambda_{eq}/I$ in terms of $L_1$ and $L_2$. Hint - make use of the fact that the total flux being stored by the two inductors is $\lambda_{eq} = \lambda_1 + \lambda_2$
b. How would you make a 4mh inductor from 2mh inductors.

9. Given two inductors connected in parallel as follows

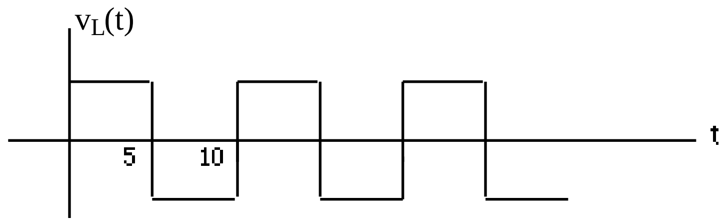



a. Express the equivalent inductance $\lambda_{eq} = L_{eq}/I$ in terms of $L_1$ and $L_2$. Hint - make use of the fact that both inductors have the same flux and so $\lambda_{eq} = \lambda_1 = \lambda_2$
b. How would you make a 5mh inductor from 2mh inductors.

10. The objective of this and the next problem is to find power and energy for inductors. What we find is that inductors, just like capacitors, store their energies. Suppose, in particular, that we have an inductor with current $i_L(t)$ as follows




and current

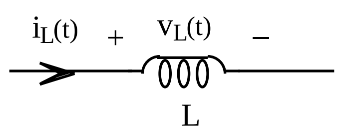



a. Explain in words how you know that $v_L(t)$ is the voltage corresponding to $i_L(t)$
b. Sketch the power $p_L(t) = v_L(t)i_L(t)$
c. How is your curve in part (b) different than power curves for resistors
d. Now sketch the energy

$$E_L(t) = \int p_L(t)dt = \int v_L(t)i_L(t)dt$$

assuming $E_L(0) = 0$

11. Now suppose we have a voltage $v_L(t)$ across an inductor as follows




that initially has no flux. Then the energy stored in the magnetic field of the inductor's coil is given by

$$E_L(t) = \int p_L(t)dt = \int v_L(t)i_L(t)dt = \int i_L(t)\left(L\frac{di_L(t)}{dt}\right)dt = \frac{1}{2}Li_L^2(t)$$

**Memorize** this result. Then make use of it to

a. Sketch $E_L$ as a function of $i_L$
b. Sketch $E_L$ as a function of $\lambda_L$
c. Show that it takes more work to increase the current through an inductor from 1ma to 2ma than from 0ma to 1ma. Why do you think this is so.

12. Sketch $v(t) = K_1e^{-1000t} + K_2e^{-2000t}$ if $v(0) < 0$ and $v'(0) < 0$