ECE 207 – FIRST ORDER RC CIRCUITS – INVESTIGATION 11
TIME CONSTANTS OF FIRST ORDER RC CIRCUITS

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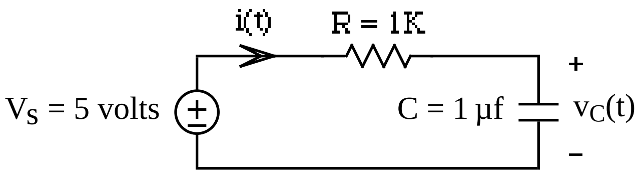
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

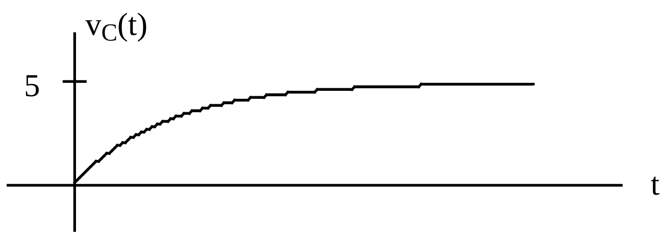
The big earth shaking observation from the last investigation is that we have to write and solve differential equations to analyze circuits containing capacitors – to determine the voltages and currents as the capacitors charge and discharge. We can no longer simply use algebra like like we did when analyzing resistor circuits. The objective of this investigation is to continue our look at the responses of first order RC circuits with constant inputs

1. We know from the previous investigation that the voltage across the capacitor in the following first order RC circuit with constant input and zero initial conditions



$v_C(0) = 0$ volts

looks as follows



with $v_C(t) = 5 - 5e^{-1000t}$ getting closer and closer to $V_S = 5$ volts as the capacitor charges up. Now, this response - like the responses of all such circuits with nonzero inputs - is the sum of two "parts"

(1) The first part - called the **forced response** - is present as a result of the input - also referred to as the forcing function. If the input wasn't there, it wouldn't be there

(2) The second part - called the **natural response** - is always present no matter what the input is

a. What would you expect are the forced and natural responses - referred to as $v_f(t)$ and $v_n(t)$ - of $v_C(t)$. Justify

b. Adding together the forced and natural responses gives us the **complete** response as follows

$$v_C(t) = v_n(t) + v_f(t) = -5e^{-1000t} + 5$$

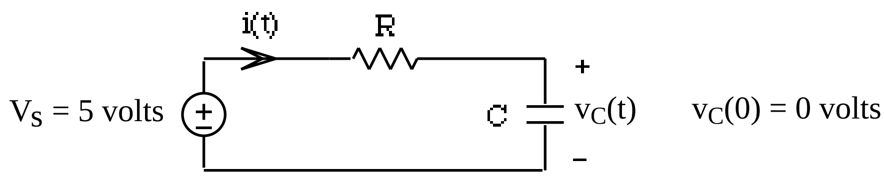
Sketch the natural, forced and complete responses of $v_C(t)$ on separate graphs. Note that when the natural response decays to zero as it does in this example, then we refer to it as the **transient** part of the response since for all practical purposes it's only around for a "short" time before **decaying** to zero

c. Once the transient part of the response dies away in a circuit like in this problem with a constant input we are left with what we refer to as the **steady state** part of the response. What is the steady state part of the response of $v_C(t)$

2. Given the first order RC circuit of Problem (1)
   a. How would you expect the size of R to affect i(t) and therefore how fast $v_C(t)$ approaches its steady state value of 5 volts. Why
   b. Sketch graphs of $v_C(t)$ for a small, medium and large value of R
   c. Now calculate and plot $v_C(t)$ for a larger resistor R = 2K and see what happens. Are your results what you expected. If not, explain what in fact is going on

3. Given the first order RC circuit of Problem (1)
   a. How would you expect the size of C to affect how fast $v_C(t)$ approaches its steady state value of 5 volts. Why.
   b. Sketch graphs of $v_C(t)$ for a small, medium and large value of C
   c. Now calculate and plot $v_C(t)$ for a larger capacitor C = 2 μf and see what happens. Are your results what you expected. If not, explain what in fact is going on

4. The objective of this problem is to get a rule of thumb for estimating how long it takes the transient responses of first order circuits like the following



to decay to zero and the circuits reach steady state. Let us begin by **defining** the **time constant** τ of such a circuit by

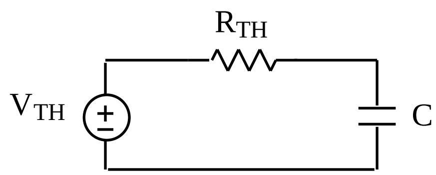
$$\tau \equiv RC = \text{RC time constant} = \text{one over the coefficient of t}$$

We then have that

$$v_C(t) = 5 - 5\,e^{-t/RC} = 5 - 5\,e^{-t/\tau}$$

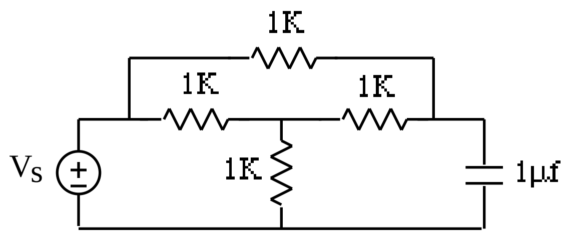
   a. Verify that RC has the units of time in seconds
   b. Set up a Table of $5\,e^{-t/\tau}$ for $t = \tau$, $t = 2\tau$, $\cdots$, $t = 5\tau$. Include not only the values of the transient part of the response at each multiple of τ but also the percentage by which it has decayed from its value at time t = 0
   c. Describe what's happening. Indicate the times $t = \tau, \cdot\cdot\cdot, 5\tau$ on a graph of $v_C(t)$
   d. For how many time constants τ would you wait before you would say the transient response has decayed and the circuit is in steady state

5. Generalizing on the result of Problem (4) we have that if we take any first order RC circuit and Thevenize it as follows



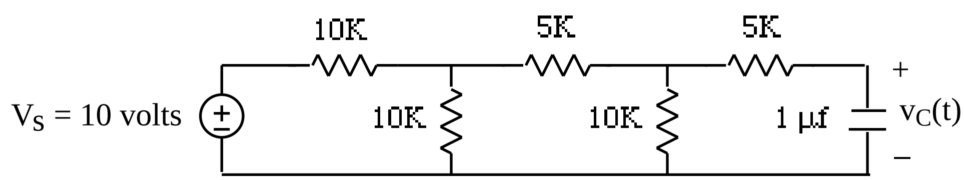
then τ for such a circuit is equal to $\tau = R_{TH}C$. **Memorize** this result for first order RC circuits. Note that most engineers consider transient responses to have decayed to zero by time 5τ

a. Find τ and the time it takes the following first order RC circuit to for all practical purposes reach steady state



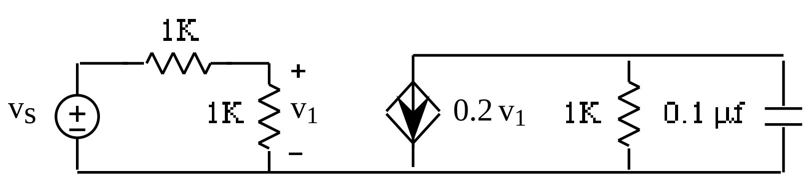
Be sure to draw the circuit you're analyzing to find R_{TH}

b. Repeat part (a) for the following circuit

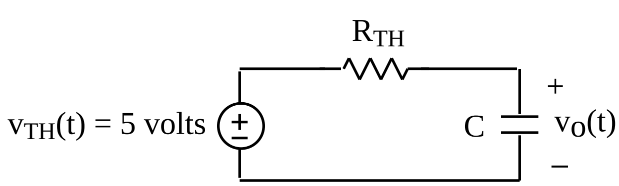


Note that it's straightforward to find R_{TH} for this circuit without having to write or solve any equations. Be sure to draw the circuit you're analyzing to find R_{TH}

6. Find the time constant τ in the following first order RC circuit containing a controlled source

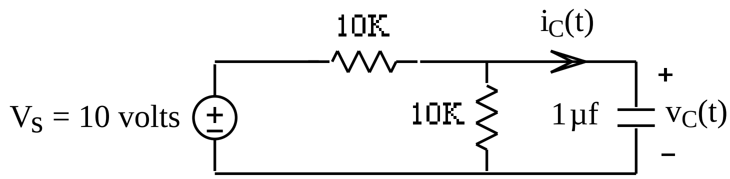


7. Given the following circuit



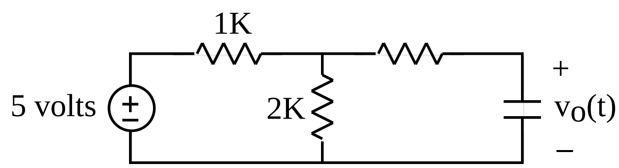
a. Sketch $v_O(t)$ if $\tau = 1$ msec
b. Sketch $v_O(t)$ if $\tau = 3$ msec
c. Describe the difference between the responses in parts (a) and (b)

8. Given the following circuit with a **constant** input



a. What's $i_C(t)$ when the circuit has reached steady state with the capacitor charged up
b. Make use of your result in part (a) to justify the fact that we can find the steady state response of an RC circuit like that above to a **constant** input by analyzing the circuit with the capacitor replaced by an open circuit.

9. Sketch the response of the following circuit



if $v_O(0) = -2$ volts and $\tau = 2$msec