# ECE 207 – FIRST ORDER RC CIRCUITS – INVESTIGATION 10
# FIRST ORDER RC CIRCUITS WITH CONSTANT INPUTS

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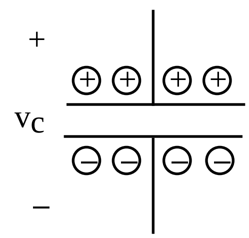
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to take our first look at what happens when we add capacitors to our circuits. We begin with first order RC circuits containing one capacitor together with resistors and independent and controlled sources. We call such circuits first order because their analysis requires the solution of a first order differential equation. One of the main results we'll be using from the last investigation is that if $i(t) > 0$ then

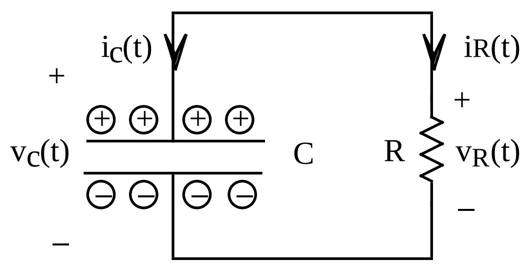
$$i(t) = C\frac{dv(t)}{dt} > 0$$

and so v(t) is increasing.

1. Suppose we take the following charged capacitor




and connect a resistor across it as follows




a. Why will current start flowing through the resistor when it is connected to the capacitor
b. Draw a picture showing the direction the equivalent positive charge is flowing in this circuit
c. Calculate $i_R(0)$ and $i_C(0)$ if R = 1K and $v_C(0)$ = 5 volts. Make sure the signs of your currents are consistent with your drawing in part (b). Note that we refer to $v_C(0)$ = 5 volts as the capacitor's **initial condition**.
d. What do you think will happen to the voltage $v_C(t)$ as the current $i_R(t)$ flows. Why. How will this, in turn, affect the magnitude of the current. Draw graphs to illustrate what you expect will happen to $i_R(t)$, the charge $q_C(t)$ and to $v_C(t) = v_R(t)$ as time passes
e. What happens to the rate at which $v_C(t)$ is changing as the magnitude of $v_C(t)$ decreases. How do you know

f. In part (c) we could use $v_C(0)$ to algebraically calculate $i_C(0)$, $i_R(0)$ and so on. But to calculate these voltages and currents for any time $t > 0$ we in general have to write and solve the differential equation $v_C(t)$.

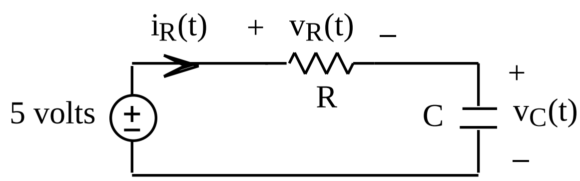
Make use of the result from the previous investigation that

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

to write and solve the differential node equation for $v_C(t)$ if $R = 1K$, $C = 1\ \mu f$ and $v_C(0) = 5$ volts. Do your analytical results agree with your original conjectured graphs. If not, explain what's going on.

g. Identify the natural and forced parts of your solution.

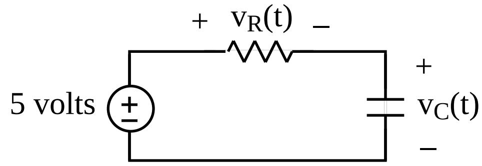
2. Now let's suppose we take the circuit of Problem (1) and add a voltage source as follows




Assuming this circuit has zero initial conditions with $v_C(0) = 0$ volts

a. What is $v_R(0)$.
b. Make use of your result from part (a) to draw a picture that shows how the equivalent positive charges are flowing in the circuit
c. Before you actually do any analysis, what do you expect will happen to $v_C(t)$ as $i_R(t)$ flows. How will this, in turn, affect $v_R(t)$. Explain. Sketch graphs to illustrate what you think will happen to $i_R(t)$, $q_C(t)$, $v_C(t)$ and $v_R(t)$ as time passes
d. Make use of $v_C(0)$ to algebraically find $i_R(0)$ and $v'_C(0)$ for $R = 1K$ and $C = 1\mu f$
e. Now write and solve the differential equation for $v_C(t)$ if $R = 1K$ and $C = 1\mu f$ for $t \geq 0$. Do your analytical results agree with your conjectured graph in part (c). If not, explain what in fact is going on.
f. Identify the natural and forced parts of your solution
g. We refer to a circuit's characteristic roots as its **natural frequencies**. **Memorize** this definition. Find the natural frequency of this circuit if $R = 1K$ and $C = 1\mu f$

3. Given the following circuit




Find $v_R(10^{-3})$ if $v_C(10^{-3}) = 2$ volts