ECE 204 - BOOLEAN ALGEBRA - INVESTIGATION 7
INTRODUCTION TO KARNAUGH MAPS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last Investigation we know that if we have a logic equation with a truth table like the following

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

then we can tell by careful inspection that F=X since F=1 when X=1 and F=0 when X=0. The objective of this Investigation is to show how we can do this "careful inspection" more easily when the Truth Tables are drawn in the form of what we call Karnaugh Maps.

It needs to be pointed out however that the "simplest design" is not necessarily simply the design with the fewest number of gates - especially in this day and age when individual gates in large integrated circuits are so inexpensive. The simplicity of the overall design, the ease of troubleshooting, the ease of modifying the design and so on are also very important considerations.

1. The **Karnaugh Map** corresponding to a Truth Table like the following

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

is simply the following

Y \ X	0	1
0	0	1
1	0	1

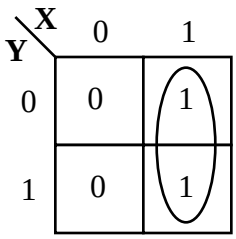
with the values in the squares equal to the values of F for the given values of X and Y. Draw the Karnaugh Map for the following Truth Table

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	0

2. Draw the Karnaugh Map for $F = X+Y$

3. Draw the Karnaugh Map for $F = X+X\cdot Y'$

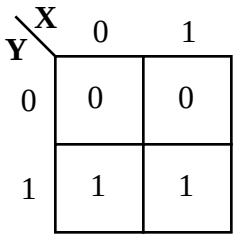
4. Karnaugh Maps are useful because they give us a straightforward way to simplify logic equations - at least those not having more than four variables. For example by *circling* the 1's in the horizontal line in the following Karnaugh Map



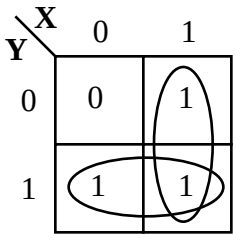
we easily see that F=1 when X=1. And so for this example we simply have that

$$F = X$$

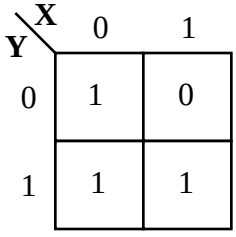
Find the equation for F having the following Karnaugh Map by first circling the "cluster" of 1's and then finding the corresponding equation



5. When a Karnaugh Map has both a row and a column of 1's as follows



we see that $F = X+Y$. Find F for the logic function with the following Karnaugh Map by first *circling* the "clusters" of 1's and then writing the corresponding equations



6. Explain how we can tell that the logic equation for the following Karnaugh Map

WX / Y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

is F=X. Be sure to circle the "cluster" of 1's.

7. Find F for the logic equation with the following Karnaugh Map

WX / Y	00	01	11	10
0	1	1	0	0
1	1	1	0	0

8. Find F for the logic equation with the following Karnaugh Map

WX / Y	00	01	11	10
0	1	0	0	1
1	0	0	0	0

Be careful - this Karnaugh Map does have a cluster

9. Up to now we've been specifying our logic functions with equations and Truth Tables. Another way to specify a logic function like F(W, X, Y) is by listing when the function is equal to one as follows

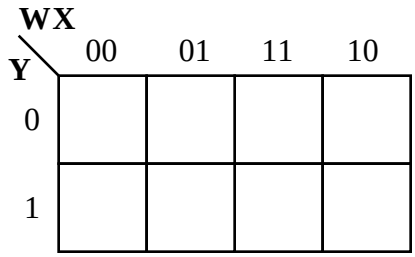
$$F = \sum_{W,X,Y} (2,6)$$

What this means is that F=1 for the minterms corresponding to 2 and 6 expressed in binary. So for this example

F(W, X, Y) = 1 for WXY = 010 and WXY = 110

Now suppose $F = \sum_{W,X,Y} (2,3,6,7)$

a. Fill in the following 3-variable Karnaugh Map for F



b. Make use of your Karnaugh Map to simplify the expression for F

10. Make use of a Karnaugh Map to simplify $F = \sum_{W,X,Y}(2,3,4,5,6,7)$

11. Make use of a Karnaugh Map to simplify $F = \sum_{W,X,Y}(0,1,4,5)$. Note that the value of X is the same for all values of WXY where F=1.

12. Up to now we've always specified the values of our logic functions F for every value of the input. But sometimes we *don't care* what the output is for some values of the input. We represent don't cares in our Truth Tables and Karnaugh Maps with *d's* as follows

WX / Y	00	01	11	10
0	0	0	d	1
1	d	1	d	0

We then separately choose values for each of the d's to make the logic equation for F as simple as possible.

a. Redraw the Karnaugh Map with values chosen for each of the d's in order to make the equation for F as simple as possible.
b. Write the logic equation for your Karnaugh Map in part (a)

13. Make use of a Karnaugh Map to simplify $F = \sum_{W,X,Y}(0,5) + d(1,3,4,6)$. Note that *d* corresponds to the don't cares. Choose the values of the don't cares to minimize the logic function. Be sure to show the values of the d's you chose.