ECE 204 - BOOLEAN ALGEBRA - INVESTIGATION 6
INTRODUCTION TO BOOLEAN ALGEBRA - PART II

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last investigation we introduced the axioms of Boolean Algebra as follows

Commutative:	$X+Y = Y+X$
	$X\cdot Y = Y\cdot X$
Distributive:	$W\cdot(X+Y) = W\cdot X + W\cdot Y$
	$W+X\cdot Y = (W+X)\cdot(W+Y)$
Identities:	$X+0 = X$; $X\cdot 1 = X$ for every X
Complements:	0 and 1 are complements of each other satisfying $X+X'=1$ and $X\cdot X'=0$

We then gave some examples to illustrate how these axioms could be used to prove the properties of switching algebra for logic circuits. The objective of this investigation is to continue using the axioms of Boolean Algebra in our development of the properties of our Switching Algebra. Our results are important because they lay the foundation for the algorithms used to systematically simplify logic equations.

1. Given the following relationship

$$F = X + X\cdot Y$$

   a. Draw a logic circuit for F
   b. Make use of your circuit in part (a) to explain in words why $F = X$. Hint - when $X=1$ then $F=1$ no matter what Y is and when $X=0$ . . .
   c. Explain how we can tell from a truth table that $F = X$

2. Given the following logic equation

$$F = X\cdot(X + Y)$$

   a. Use AOI gates to draw a logic circuit for this equation
   b. Make use of your logic circuit in part (a) to explain in words why $F = X$
   c. Explain how we can tell from a truth table that $F = X$

3. Given the following logic equation

$$F = X\cdot Y + X\cdot Y'$$

   a. Use AOI gates to draw a logic circuit for this equation
   b. Make use of your logic circuit in part (a) to explain in words why $F = X$
   c. Explain how we can tell from a truth table that $F = X$

4. From Problems (1)-(3) we see that we can simplify logic equations by careful examination of truth tables. Make use of your observations to find simplified equations for F in each of the following

a.

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0

b.

W	X	Y	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

c.

W	X	Y	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

d.

W	X	Y	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

5. The objective of this problem is to introduce the first of the two results from **DeMorgan's Theorem** as follows

$$(X + Y)' = X' \cdot Y'$$

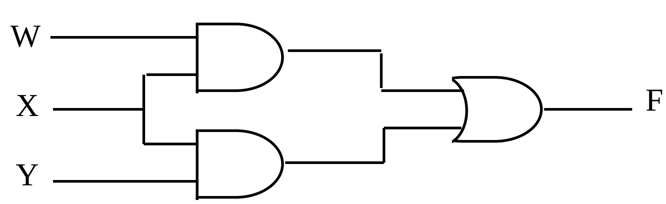
Memorize this result. Then
   a. Use AOI gates to draw logic circuits for $(X + Y)'$ and for $X' \cdot Y'$
   b. Describe in words why $(X + Y)' = X' \cdot Y'$
   c. Make use of a truth table to show that $(X + Y)' = X' \cdot Y'$

6. The objective of this problem is to introduce the second form of DeMorgan's Theorem as follows

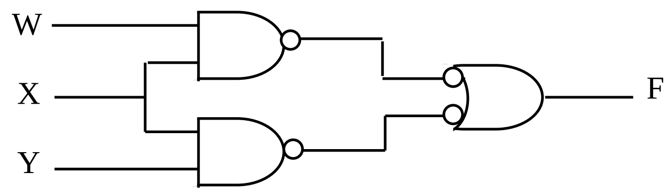
$$(X \cdot Y)' = X' + Y'$$

Memorize this result. Then
   a. Use AOI gates to draw logic circuits for $(X \cdot Y)'$ and for $X' + Y'$
   b. Describe in words why $(X \cdot Y)' = X' + Y'$
   c. Make use of a truth table to show that $(X \cdot Y)' = X' + Y'$

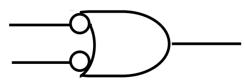
7. The objective of this problem is to use DeMorgan's Theorem to illustrate how AND-OR circuits can be realized with NAND gates. Given the following AND-OR circuit



   a. Explain why the AND-OR circuit is equivalent to the following circuit



b. Use DeMorgan's Law to show that the following can be realized with a NAND gate



c. Make use of your results in parts (a) and (b) to realize the whole circuit with NAND gates
d. Make use of your result in parts (a)-(c) to explain how AND-OR circuits can be realized with NAND gates. **Memorize** your results.

8. The objective of this and the next problem is to introduce duality. From the results of Problems (1) and (2) as follows

$$X + (X\cdot Y) = X \quad \text{and} \quad X\cdot(X + Y) = X$$

we see that we can obtain one from the other by simply replacing the pluses by dots and the dots by pluses. We say that these two expressions are **duals** of each other. Find the dual of

$$(X + Y)' = X'\cdot Y'$$

9. In Problem (8) we introduced what we mean by the dual of a logic function. More completely the **dual** of a logic function can be obtained by

   (1) Replacing every + by a · and every · by a +
   (2) Replacing every 0 by a 1 and every 1 by a 0

   The importance of duality is that it can be shown that if a logic function is true then so is its dual. We call this the **Principle of Duality**. **Memorize** this result. Then use it to show how the form of DeMorgan's Theorem in Problem (6) as follows

$$(X\cdot Y)' = X'+ Y'$$

   can be obtained from the form of DeMorgan's Theorem in Problem (5) as follows

$$(X + Y)' = X'\cdot Y'$$

10. When we express F as a sum of minterms we say F is in standard **SOP (Sum of Product)** form. **Memorize** this definition. Then express each of the following in standard SOP form. Note that SOP problems can always be done with Boolean Algebra but that truth tables are more foolproof.

    a. $F(X,Y) = X + X'\cdot Y$
    b. $F(X,Y) = (X + Y)'$
    c. $F(X,Y) = X + (X\cdot Y)'$
    d. $F(X,Y) = ((X\cdot Y)' + Y)'$