ECE 204 - BOOLEAN ALGEBRA - INVESTIGATION 5
INTRODUCTION TO BOOLEAN ALGEBRA - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the first four investigations we've shown how to analyze logic circuits with equations like the following

$$F = W \cdot X + Y$$

What's particularly significant about these equations is that they're *algebraic* - all we have to do to obtain the value of F is substitute in the values of W, X and Y and then evaluate $W \cdot X+Y$. The objective of this and the next investigation is to develop the basic properties of these equations which together form what we call *switching algebra*.

1. Why do we refer to logic equations like the following

$$F = W \cdot X + Y$$

as algebra equations. Note that we refer to the terms in logic equations like W and X as **literals**. **Memorize** this definition.

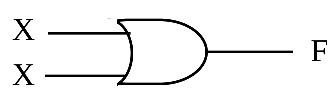
2. The objective of this problem is to show how we can use truth tables to prove two properties of *switching* algebra that illustrate how switching algebra is different from *regular* algebra.
   a. First show that

$$F = X + X = X$$

   by completing the following truth table

X	F

   Hint - make use of the truth table of an OR gate as follows



   with both inputs equal to X
   b. Now show that

$$F = X \cdot X = X$$

   HInt - make use of the truth table of an AND with both inputs equal to X

3. From the results of Problem (2) we see that the switching algebra of logic circuits is clearly different from *regular* high school algebra where $X+X=2X$ and $X \cdot X=X^2$. But the switching algebra of our logic circuits does turn out to have exactly the same properties as **Boolean Algebra**. Boolean Algebra was developed by the mathematician and logician George Boole in the 1800's to show that the results of formal mathematical logic could be obtained from the

manipulation of algebra equations with a special set of axioms we now refer to as the axioms of Boolean Algebra. The objective of this and the next problem is to "show" that our switching algebra is a Boolean Algebra by showing that it satisfies the axioms of Boolean Algebra.

The first requirement of a Boolean Algebra is that it have two operations and that they be communicative and distributive. For our switching algebra the two operations are AND and OR which we denote in our equations with dots (·) and pluses (+).

a. Make use of truth tables to show that both operators are commutative as follows

$$X+Y = Y+X \quad \text{and} \quad X \cdot Y = Y \cdot X$$

b. Make use of truth tables to show that both operators are distributive over each other as follows

$$W \cdot (X+Y) = W \cdot X + W \cdot Y \quad \text{and} \quad W+X \cdot Y = (W+X) \cdot (W+Y)$$

4. The second requirement of a Boolean Algebra is that it contain two unique elements 0 and 1 that behave much like 0 and 1 in "regular" algebra with

$$X+0=X \quad \text{and} \quad X \cdot 1=X \quad \text{for all } X$$

Since we have only two elements L and H in our switching algebra one must be 0 and the other 1. When we make the assignment

$$1 \leftrightarrow H \quad \text{and} \quad 0 \leftrightarrow L$$

we call it **positive logic**. And when we make the assignment

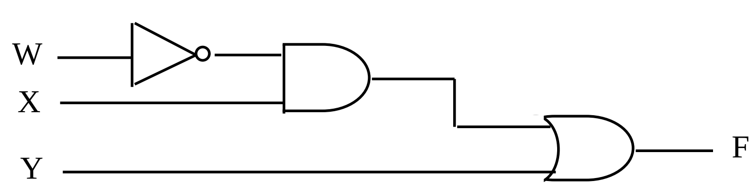
$$0 \leftrightarrow H \quad \text{and} \quad 1 \leftrightarrow L$$

we call it **negative logic**. **Memorize** these defintions. We will use **positive logic** throughout this class.

a. Write the following truth table in terms of 1's and 0's for our positive logic

X	**Y**	**F**
L	L	L
L	H	H
H	L	H
H	H	L

b. Write a truth table for the following logic circuit in terms of 1's and 0's



c. Show that X+0=X. Hint - complete the following truth table

X	X + 0

d. Show that $X \cdot 1=X$

5. Make use of the fact that $0'=1$ and $1'=0$ to find
   a. $V = 0+0'$
   b. $X = 0\cdot0'$
   c. $Y = (0+1)'$

6. The third requirement of a Boolean Algebra is that every element X have a complement X' satisfying the following relationships

$$X+X'=1 \qquad \text{and} \qquad X\cdot X'=0$$

Verify that these two relations are satisfied for 0 and its complement $0'=1$. And then for 1 and its complement $1'=0$.

7. Combining the results of the last group of problems we have that our switching algebra is a Boolean Algebra since it satisfies the following axioms

Commutative:	$X+Y = Y+X$
	$X\cdot Y = Y\cdot X$
Distributive:	$W\cdot(X+Y) = W\cdot X + W\cdot Y$
	$W+X\cdot Y = (W+X)\cdot(W+Y)$
Identities:	$X+0 = X$; $X\cdot 1 = X$ for every X
Complements:	0 and 1 are complements of each other satisfying $X+X'=1$ and $X\cdot X'=0$

Now that we have the axioms of Boolean Algebra we can use them to derive results for all Boolean Algebras. For example to prove for all Boolean Algebras

$$X + 1 = 1 \quad \text{for all } X$$

we can proceed as follows

$1 = X + X'$	by complements
$= X + X'\cdot(1)$	by identities
$= (X+X')\cdot(X+1)$	by distributive
$= (1)\cdot(X+1)$	by complements
$1 = X + 1$	by identities

Write out the similar proof showing that

$$X\cdot 0 = 0 \quad \text{for all } X$$

8. Even though the logic equation $F = V \; W + X' \; Y$ is an equation representing AND's and OR's as follows F = (V AND W) or (NOT X AND Y) we refer to the right hand side of the equation as a *sum of products* just like we do in *regular* algebra.
   a. Write out a logic equation equal to a sum of products
   b. Write out a logic equation equal to a product of sums

9. When people talk about logic equations they often use the terms True (T) and False (F) instead of High and Low. They say for example that

$$Z = X\cdot Y$$

is True if both X AND Y are True. Write the equation for Z if Z is True when

(W OR X is True) AND Y is True