ECE 130 - INDUCTION AND RECURSION - INVESTIGATION 8
INTRODUCTION TO RECURSION

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last Investigation we reviewed induction. The main objective of this Investigation is to introduce recursion - a method of implementing algorithms "inductively".

1. Let us begin with the problem of how best to generate a table of factorials $n!$. The brute force way is to simply calculate $n!$ for each desired n. But from the following expression for $n!$

   $$n! = n(n-1)!$$

   we see that we can calculate $n!$ from the previous factorial $(n-1)!$. Make use of this observation to obtain $n!$ for n=0 to n=5. Put your results in a Table. Remember that $0! = 1$

2. Formalizing the result from Problem (1) we have that if $F(n) = n!$ then

   $$F(n) = nF(n-1) \quad \text{for} \quad n \geq 1$$

   with $F(0) = 0! = 1$. We call this a **recursive equation** for $F(n)$. What's the advantage of recursion when we want to generate a Table of values like those for $n!$

3. The purpose of this and the next two problems is to write recursive equations for Fibonacci numbers - solutions to the following problem introduced by Fibonacci in 1202. "How many pairs of rabbits can be produced from a single pair of rabbits in one year?" Assume that the initial pair of rabbits, born in January, has its first pair of babies two months later in March and subsequently has exactly one pair of new babies every month thereafter. Similarly its babies born in March start having their single pairs of babies in May and so on. Set up a Table showing what's going on every month - with columns for how many pairs of rabbits there are, how many babies they're having and so on.

4. Make use of your Table in the last problem to find a recursive expression for the number of rabbits $F(n+1)$ after $n+1$ months. Set up a Table to verify that your expression works.

6. The sequence of numbers generated in Problem (5) by the recursive relationship

   $$F(n+1) = F(n) + F(n-1)$$

   with $F(1) = F(2) = 1$ is what we call the **Fibonnaci numbers.** Write out the Fibonnaci sequence from n=1 to n=7

7. Come up with the recursive equation for a recursion problem of your own.

8. The objective of this problem is to point out the difference between iterative algorithms and recursive algorithms for solving recursive equations. Suppose in particular that we want to find

   $$S(n) = 2S(n-1) \qquad S(1) = 2$$

   for $n \geq 2$. Given the following iterative and recursive algorithms for calculating S(n)

```
S_iterative(integer n)
A=2
i=2
while i≤n do
      A = 2*A
      i = i+1
end while
Return A
```

```
S_ recursive(integer n)
S_recursive(1) = 2
Return 2*S_recursive(n-1)
```

a. Explain how the iterative algorithm calculates S(n) - and in particular why it's not a recursive algorithm
b. Explain how the recursive algorithm calculates S(n) - and in particular why it is a recursive algorithm
c. Why are the memory requirements of the recursive algorithm more than that for the iterative algorithm