ECE 130 - INDUCTION AND RECURSION - INVESTIGATION 7
PROOF BY INDUCTION

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last two Investigatiopns we introduced algorithms for searching and sorting and got an idea of how to calculate their complexities. In this Investigation we review proof by induction and then in the next several Investigations extend our results to the development of recursive algorithms.

1. Suppose we want to show that a given polynomial P(n) is odd for all integers n > 0. One way we can do it is as follows

   (1) First show that P(1) is odd

   (2) And then show that if P(n) is odd then P(n+1) is odd

   This is referred to as the **First Principle of Mathematical Induction**. Make use of the First Principle of Mathematical Induction to show that if n is odd then n^2 is odd. Note that positive odd integers can be written as $n = 2k + 1$ for k = 0, 1, 2, . . .

2. Make use of induction to show that $2 + 4 + 6 + \cdots + 2n = n(n + 1)$

3. The **Second Principle Of Mathematical Induction** is a variation on the First Principle of Mathematical Induction as follows. Suppose we again want to show that a given polynomial P(n) is odd for all integers n > 0. We can do this as follows

   (1) First show that P(1) is odd

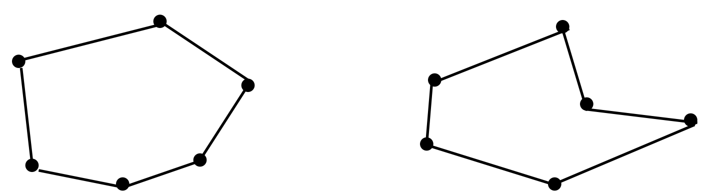
   (2) And then show that if P(1), P(2), ..., P(n) are odd then P(n+1) is odd

   Explain why these two principles of induction are equivalent

4. Prove that any integer n ≥ 2 can be written as a sum of 2's and 3's

5. Prove that any integer n ≥ 12 can be written as a sum of 4's and 5's

6. Prove that the sum of the interior angles of n-sided "simple" polygons like the following



   is (n-2)180° for n ≥ 3

7. Given the following sum

$$A = 1(1!) + 2(2!) + \cdots + n(n!)$$

   a. Find an expression for A as the difference of two factorials. Explain how you got your result
   b. Make use of induction to prove your result in part (a)