ECE 130 - INTRODUCTION TO ALGORITHMS - INVEST 6
INTRODUCTION TO THE COMPLEXITY OF ALGORITHMS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last Investigation we introduced several searching and sorting algorithms. To decide which algorithm to use in any given application we need to consider a number of factors including speed, complexity and memory requirements. The objective of this Investigation is to compare different search and sorting algorithms by counting the number of comparisons.

1. We begin with some finding of upper bounds on expressions. Show that
   a. $x^2 + 2x + 1 < 4x^2$ when $x \geq 1$
   b. $x^2 + 2x + 1 < 3x^2$ when $x \geq 2$

2. Find k in the expression $x^2 + 2x + 1 < 2x^2$ when $x \geq k$

3. The objective of this problem is to introduce **BIG-O** notation with an example. We say that a function f(x) **is of order** x^2 - which we denote by $O(x^2)$ - if there exists constants C and k such that

$$|f(x)| \leq Cx^2 \quad \text{for all} \quad x > k$$

   a. Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$
   b. Find a function $f(x)$ of $O(x)$
   c. Find a function $f(x)$ of $O(x^3)$

4. Write out a definition for the more general case of what it means for a function f(x) to be on the order of a function g(x) - to be $O(g(x))$

5. Suppose the number of computations required by algorithm A is $O(n^2)$ and the number required by Algorithm B is $O(n)$. All else being equal, which one would you use. Why

6. Show that $\log(n!)$ is $O(n \log n)$

7. How many comparisons are required to do a bubble sort of n=5 numbers. Note that after the first sort the largest number is in its proper location. And so on.

8. Generalizing on the result of Problem (7) it can be shown that in the worst case a bubble sort takes $\frac{(n-1)n}{2}$ comparisons. What is the order of the bubble sort.

9. Given an insert sort with n=5 numbers
   a. What's the maximum possible number of comparisons that would have to be done
   b. What's the minimum possible number of comparisons that could be done

10. Generalizing on the result of Problem (9) it can be shown that in the worst case the most comparisons that an insert sort would require is $\frac{n(n+1)}{2} - 1$. What is the order of insert sort

11. Given an algorithm, what would you expect is meant by
    a. Worst-case complexity
    b. Average-case complexity
    c. Which would you expect is more important in real time applications

12. Given that

$$\lfloor x \rfloor = \text{Largest integer} \le x$$

$$\lceil x \rceil = \text{Smallest integer} \ge x$$

Memorize this notation. And then find

a. $\left\lfloor \frac{8}{3} \right\rfloor$

b. $\left\lceil \frac{8}{3} \right\rceil$