ECE 130 - THE VERY BASICS - INVESTIGATION 4
PROOF BY CONTRADICTION

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last Investigation we reviewed direct proofs and proofs by contraposition. The main objective of this Investigation is proof by contradiction.

1. We begin with some review problems. Prove or disprove (with a counterexample) the claim that

$$\text{if } n \text{ is prime then } n+4 \text{ is prime}$$

2. Make use of contraposition to show that if $x<0$ then $x-1<0$

3. In contraposition we prove $A \rightarrow B$ by showing that $B' \rightarrow A'$. In **proof by contradiction** we prove

$$A \rightarrow B$$

   by proving that $A \quad B'$ (A and B') is not possible - that they cannot both be true at the same time. Explain in your own words the difference between proof by contraposition and proof by contradiction.

4. A classic example of proof by contradiction is showing that $x=\sqrt{2}$ is irrational
   a. What are A and B in this theorem
   b. What is B'
   c. Now if B' is true then $x=\sqrt{2}$ is rational and so can be expressed as a ratio as follows

$$\sqrt{2}=\frac{m}{n}$$

      reduced to lowest form with no common factors. Given this assumption solve for m^2
   d. Make use of your result in part (c) to show that m is even. Hint - if the square of m is even then . . .
   e. Now if m is even it can be written as $m=2p$ for some integer p. Make use of this result to show that n is even just like you showed that m is even in part (d)
   f. Pulling things together we have that m and n are both even. Therefore we have a contradiction that m and n have no common factors. Explain why this proves that $A \rightarrow B$. That $\sqrt{2}$ is irrational

5. From the previous problem we see that proof by contradiction is useful for proving $A \rightarrow B$ when B says something is not true. Prove that $\sqrt[3]{2}$ is not rational

6. Come up with your own example of a proof by contradiction