ECE 130 - THE VERY BASICS - INVESTIGATION 3
DIRECT AND CONTRAPOSITIVE PROOFS

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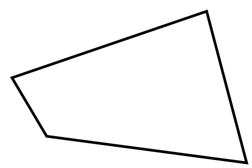
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last two Investigations we reviewed sets. The objective of this and the next Investigation is to review different proof techniques. The objective of this Investigation is to review direct proofs and proof by contrapositive.

1. Let us begin with a simple direct proof from geometry. Make use of the fact that the sum of the angles of a triangle is 180˚ to prove that if P is a 4-sided polygon P like the following

   then the sum of its angles is 360˚

2. Proofs like those in Problem (1) are called **direct proofs** of the form *if A then B*. What makes them direct is that we make use of the assumption that A is true to show B. What are *A* and *B* in the theorem of Problem (1)

3. If we have a theorem of the form *if A then B* then we write $A \rightarrow B$ and say *A implies B*. **Memorize** this notation and terminology. Make up and do your own direct proof. Identify A and B

4. When we do proofs for discrete structures we often need to break the proof into separate cases. The classic example is the four color problem which states that any map drawn on a plane can be drawn with at most four colors without any ambiguity over borders. The theorem was in fact proved with the help of a computer that checked about 2000 different cases. Let us do a simpler example. Show that the squares of the integers 1 to 5 are even when n is even and odd when n is odd by calculating n^2 for each case - for each integer.

5. Let us now generalize on the result of Problem (4)
   a. Make use of the fact that even numbers are of the form 2n to show that the square of an even number is even
   b. Make use of the fact that odd numbers are of the form 2m+1 to show that the square of an odd number is odd
   c. Show that the product of an even number times an odd number is even

6. Along the same lines as Problem (5)
   a. Show that the product of any two consecutive numbers is even
   b. Show that the product of any three consecutive numbers is even

7. Now we do something a little different. Make use of the fact that the square of an even integer is even to show that if n^2 is odd then n must be odd. Hint - if n is even then . . .

8. Proofs like the one in Problem (7) are called proof by **contraposition** - we show and then

make use of the fact that if the conclusion isn't true then the assumption can't be true. More formally we prove

$$A \rightarrow B \text{ by showing that } B' \rightarrow A'$$

Memorize what proof by contraposition is and then explain in your own words why the truth of $B' \rightarrow A'$ implies the truth of $A \rightarrow B$. Note that $B' \rightarrow A'$ is called the **contrapositive** of $A \rightarrow B$

9. Use contraposition to show that if $x > 0$ then $x + 1 > 0$

10. Make up your own example of a proof by contraposition

11. The objective of this problem is to introduce what we mean by the **converse** of a theorem. The converse of

$$A \rightarrow B \text{ is } B \rightarrow A$$

   a. Come up with an example where both a theorem and its converse are true
   b. Come up with an example where a theorem is true but not its converse. In particular give a **counterexample**

12. If both $A \rightarrow B$ and its converse $B \rightarrow A$ are true then we say B is true if and only if (**iff**) A is true.
   a. Why are we able to say *only if*
   b. Verify the *only if* part for your example in Problem (11a)