ECE 130 - MATRICES - INVESTIGATION 25
INTRODUCTION TO MATRICES - PART III

WINTER 2004 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

From the last two Investigations we know how to put linear equations in matrix form, we know how to add, subtract and multiply matrices, and we know how to use the inverses of matrices to solve matrix equations. The main objective of this Investigation is to get some more practice working with matrices and to use them to determine if a graph is connected.

1. Multiply together the following matrices

   a. $\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

   b. $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 & 4 \end{pmatrix}$

2. Do an example with 2×2 matrices to illustrate the **associative law** for matrix multiplication as follows

$$(AB)C = A(BC)$$

3. Do an example with 2×2 matrices to illustrate the **distributive law** for matrices as follows

$$A(B + C) = AB + AC$$

4. As we saw in the last Investigation "all" we have to do to solve a set of linear equations as follows

$$Ax = b$$

   is find the inverse of A and then calculate

$$x = A^{-1}b$$

   The problem of course is to find A^{-1}. Determinants work fine for two or three variables but that's about it - especially for people. Computers typically use some form of Gaussian elimination to in affect calculate A^{-1}.

   In this problem we work with the following general equation for the inverse of a 2×2 matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \Leftrightarrow \quad A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

   for when $a_{11}a_{22} - a_{12}a_{21} \neq 0$

   a. Verify that $AA^{-1} = I$

b. Find A^{-1} of $A = \begin{pmatrix} 2 & -3 \\ 5 & 6 \end{pmatrix}$

c. Make use of A^{-1} to solve the following simultaneous equations

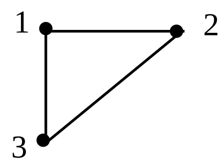
$$\begin{aligned} 3x_1 + 2x_2 &= 2 \\ -x_1 + 3x_2 &= 5 \end{aligned}$$

5. When we solve a pair of linear equations we're finding the intersection of two lines. But not all lines intersect. Do an example to illustrate that when two lines are parallel then their coefficients satisfy

$$a_{11}a_{22} - a_{12}a_{21} = 0$$

and so A^{-1} doesn't exist. We then say A is **singular**

6. Another interesting application of matrices has to do with finding paths in graphs. If A is the adjacency matrix of a graph G as follows



then the (i, j)'th entry of A^r is the number of different paths of length *r* from v_i to v_j. Note in particular that vertices can be visited more than once in these paths

a. Find the adjacency matrix A
b. Find A^2
c. Verify that the (i, j) entries in A^2 are equal to the number of distinct paths from i to j of length two

7. How can we make use of the result in Problem (6) to find the shortest path between two vertices in a graph. Illustrate with an example

8. How can we make use of the result in Problem (6) to determine if a graph is connected. Illustrate with examples of graphs that are connected and are not connected

9. Make use of your result in Problem (8) to determine if the graph *G* with the following adjacency matrix is connected

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$