ECE 130 - MATRICES - INVESTIGATION 23
INTRODUCTION TO MATRICES - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

As we saw in Investigation 15 matrices matrices are simply arrays of numbers like the following

$$B = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$$

The main objective of this Investigation is to show how sets of linear equations can be put in matrix form to facilitate solving them

1. We put linear equations like the following

$$\begin{aligned} 3x_1 + 2x_2 &= 4 \\ x_1 - 5x_2 &= 2 \end{aligned}$$

   in matrix form as follows

$$\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

   so we can more easily work with them - come up with general expressions for their solution as well as more easily write algorithms for computer solution.

   Note in particular that

$$\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} = \text{Coefficients of the unknowns } x_1 \text{ and } x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \text{Unknowns we're solving for}$$

   a. Put the following linear equations in matrix form

$$\begin{aligned} 4x_1 - 3x_2 &= 3 \\ 2x_1 - 6x_2 &= -1 \end{aligned}$$

   b. Put the following linear equations in matrix form

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 3 \\ x_1 \qquad - 4x_3 &= 0 \\ 5x_1 + 3x_2 + 6x_3 &= -4 \end{aligned}$$

   c. Write out the equations for

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & -4 \\ 5 & 3 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

2. If we now take a set of linear equations in matrix form like the following

$$\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

and let

$$A = \begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

we can then write our matrix equation as

$$Ax = b$$

This form is really nice because a whole set of linear equations look like one simple equation

a. Find A and b for the following set of linear equations

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 3 \\ x_1 \qquad - 4x_3 &= 0 \\ 5x_1 + 3x_2 + 6x_3 &= -4 \end{aligned}$$

b. Find the linear equations with

$$A = \begin{pmatrix} 4 & 7 \\ -1 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$$

3. We call a matrix with two rows and two columns like the following

$$A = \begin{pmatrix} 4 & 7 \\ -1 & 2 \end{pmatrix}$$

a 2×2 (2 by 2). More generally we say a matrix with m rows and n columns is an $m \times n$ matrix. And we call $m \times n$ the **dimension**. Write out examples of matrices with the following dimensions

a. 3×3
b. 2×3
c. 3×2
d. 1×3
e. 3×1

4. Would you call a $1 \times n$ matrix a row matrix or a column matrix. Why. Illustrate with an example

5. Would you call a $n \times 1$ matrix a row matrix or a column matrix. Why. Illustrate with an example

6. What would you call an $n \times n$ matrix

7. For a general matrix like the following 2×2 square matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

the subscripts tell us the row and column of each of the entries. In particular

$$a_{jk} = \text{entry in row j and column k}$$

Memorize this relationship

   a. Write out the general form for B equal to a matrix
   b. Where is a_{52} located in a 6×6 matrix

8. Solving matrix equations like those in this Investigation can be relatively simple or it can involve a fair amount of computation. In this and the next problem we'll look at cases when the computations are very easy. Given the following matrix equation

$$\begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

   a. Solve for x_1 and x_2
   b. What makes this matrix equation easy to solve
   c. Note that matrices like A in our equation is called a **diagonal** matrix. Why do we call it a diagonal matrix
   d. Make up your own example of a matrix equation with a 3×3 diagonal A matrix that's similarly easy to solve

9. Here's another example of matrix equation that's relatively easy to solve

$$\begin{pmatrix} 3 & 0 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

   a. Solve for x_1 and x_2
   b. What makes this matrix equation easy to solve
   c. Find out what we call the matrix A in this equation
   d. Make up your own example of a matrix equation with a 3×3 A matrix like the one in this problem that's similarly easy to solve