ECE 130 - THE VERY BASICS - INVESTIGATION 2
SET OPERATIONS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last Investigation we reviewed what sets are and how to specify them with lists, statements and Venn Diagrams. The objective of this Investigation is to review and introduce operations on sets.

1. The objective of this problem is to review the union of sets

   *The **union** of two sets A and B, denoted by $A \cup B$, is the set that contains all the elements that are in either A or B*

   a. Write out in words the symbolic definition of union $A \cup B = \{x \mid x \in A \vee x \in B\}$ where $\vee$ means *or*
   b. Find $C = A \cup B$ when $A = \{1,2,3\}$ and $B = \{2,3,4\}$. Illustrate with a Venn Diagram

2. The objective of this problem is to review the intersection of sets

   *The **intersection** of two sets A and B, denoted by $A \cap B$, is the set that contains all the elements that are in both A and B*

   a. Write out in words the symbolic definition of intersection $A \cap B = \{x \mid x \in A \wedge x \in B\}$ where $\wedge$ means *and*
   b. Find $C = A \cap B$ when $A = \{1,2,3\}$ and $B = \{2,3,4\}$. Illustrate with a Venn Diagram
   c. What do we mean when we say two sets are **disjoint**.
   d. Give an example of two disjoint sets

3. The objective of this problem is to calculate the cardinalities of the unions and intersections of sets

   a. What is the cardinality of A intersect B as follows $|A \cap B|$ when A and B are disjoint
   b. Do an example to illustrate the result $|A \cup B| = |A| + |B| - |A \cap B|$ when A and B are not disjoint
   c. Do an example to illustrate the result $|A \cup B| = |A| + |B| - |A \cap B|$ when A and B are disjoint
   d. Explain in words why our equation $|A \cup B| = |A| + |B| - |A \cap B|$ is true - why in particular we subtract $|A \cap B|$. Note that this result is referred to as the **principle of inclusion-exclusion**

4. The objective of this problem is to review the difference of sets

   *The **difference** of two sets A and B, denoted by $A - B$, is the set that contains all the elements that are in A but not B*

   a. Write out in words the symbolic definition of intersection $A - B = \{x \mid x \in A \wedge x \notin B\}$
   b. Find $C = A - B$ when $A = \{1,2,3\}$ and $B = \{2,3,4\}$. Illustrate with a Venn Diagram

5. The objective of this problem is to define the complement of a set A

The ***complement*** *of a set A, denoted by* A'*, is the set that contains all the elements in the universal set U that are not in A*

a. Write out in words the symbolic definition of intersection $A - B = \{x \mid x \quad A\}$
b. Find $C = A'$ when $A = \{1,2,3\}$ and $U = \{1,2,3,4,5,6\}$. Illustrate with a Venn Diagram

6. The objective of this problem is to introduce **DeMorgan's Theorem**. Given the following sets A and B

$$A = \{1, 2, 3\} \qquad B = \{3, 4, 5\}$$

a. Verify Form 1 of DeMorgan's Theorem as follows $(A \quad B)' = A' \cap B'$. **Memorize** this relationship
b. Make use of a Venn Diagram to illustrate Form 1 of DeMorgan's Law
c. Verify Form 2 of DeMorgan's Theorem as follows $(A \cap B)' = A' \quad B'$. **Memorize** this relationship
d. Make use of a Venn Diagram to illustrate Form 2 of DeMorgan's Law

7. Do examples to illustrate the following relations. Illustrate with Venn Diagrams

a. $A \cap (B \quad C) = (A \cap B) \quad (A \cap C)$
b. $A \quad (B \cap C) = (A \quad B) \cap (A \quad C)$

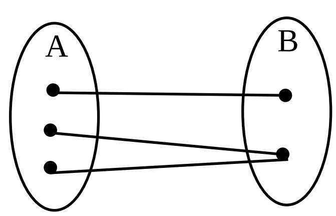
8. Now that we know how to work with sets and subsets by hand we need a nice convenient way to represent them in computers. One way is to choose an arbitrary order for the universal set like the following

$$A = \{a, 1, b, c, 2, 3\}$$

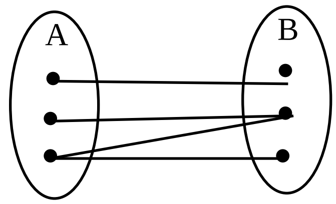
and then use **bit strings** to represent the sets. Suppose for example that $A = \{a, c, 3\}$ then the corresponding bit string for A is 100101

a. What does a 1 in the bit string mean
b. What does a 0 in the bit string mean
c. What are the elements in C = 110101   101010
d. What are the elements in D = 110101   101010

9. The objective of this and the rest of the problems in this Investigation is to make use of sets to review the properties of functions. Make use of the fact that the following diagram is for a function from A to B



while this is not a function from A to B



to explain how functions are different from general relations.

10. Draw a diagram like in Problem (9) for a function that is into but not onto

11. Draw a diagram like in Problem (9) for a function that is onto but not one-to-one

12. Draw a diagram like in Problem (9) for a function that is onto and one-to-one

13. Which of the functions in Problems (10)-(12) has an inverse