ECE 130 - TREES - INVESTIGATION 19
INTRODUCTION TO TREES

WINTER 2004 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last group of Investigation we introduced graphs and some of their applications. The objective of this and the next two Investigations is to introduce a special kind of graph called a tree. Trees are useful in a number of applications including finding iterms in lists.

1. Given that the following graphs are trees

while these are not

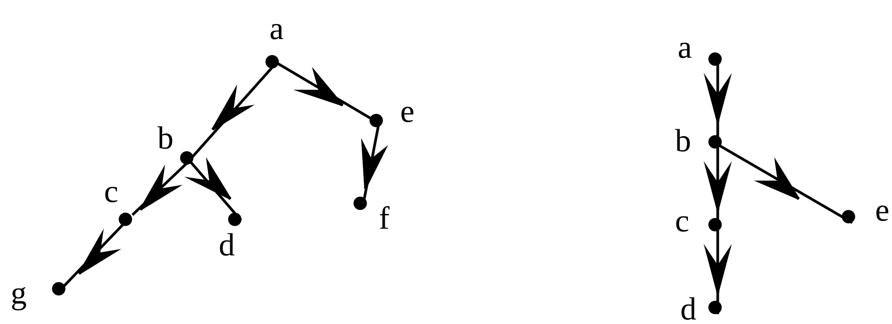
Write in your own words the differences between graphs that are trees and graphs that are not

2. From the previous problem we know that trees are connected graphs that do not contain any circuits
   a. Draw two graphs that are trees
   b. Draw two graphs that are not trees

3. Show that if G is a tree then there is a unique path between every pair of vertices

4. Show that if a graph G has a unique path between every pair of vertices then it's a tree

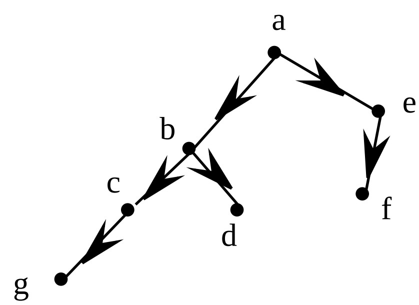
5. The following are examples of rooted trees



   a. What, in your own words, are rooted trees

b. Draw a rooted tree
c. Draw a tree that is not rooted

6. Given a rooted tree as follows



a. What would you expect is meant by the (single) **parent** of a vertex
b. What is the parent of vertex *c*
c. What vertex does not have a parent
d. What would you expect is meant by the **children** of a vertex.
e. What are the children of vertex *b*
f. The **internal vertices** of a tree are all the vertices that have children. What are the internal vertices of the above tree
g. A vertex is a **leaf** if it has no children. What vertices above are leaves
h. What would you expect is meant by the **descendants** of a vertex
i. Find all the descendants of vertex *b*

7. A rooted tree is called **m-ary** if no internal vertex has more than m children. A rooted tree is called full m-ary if every internal node has exactly m children
   a. Draw a 3-ary tree with at least six vertices that is not full 3-ary
   b. Draw a complete 3-ary tree with at least six vertices

8. Illustrate the fact that a **full m-ary** tree with i internal vertices has a total of $n = mi + 1$ vertices

9. Show that a full m-ary tree with i internal vertices has a total of $n = mi + 1$ vertices

10. Given a tree T
    a. What is meant by the **depth** of the tree
    b. Draw a tree of depth four