ECE 130 - GRAPHS - INVESTIGATION 18
SHORTEST PATH PROBLEMS

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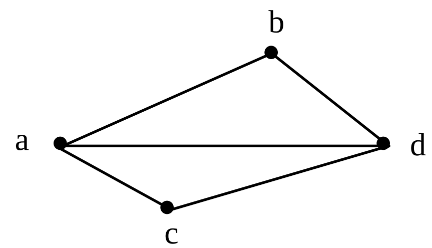
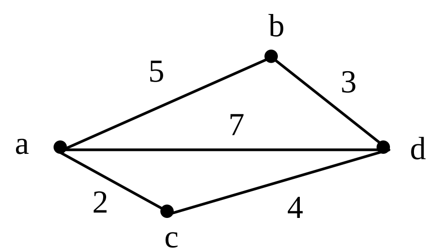
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last Investigation we talked about finding paths and circuits in graphs. In this Investigation we introduce the problem of how to find the shortest paths in what are called weighted graphs.

1. The objective of this first problem is to introduce what we mean by **weighted graphs**. Suppose we have a graph as follows



with the vertices representing cities and the edges highways between them. If we now add the lengths of the highways to the graph as follows

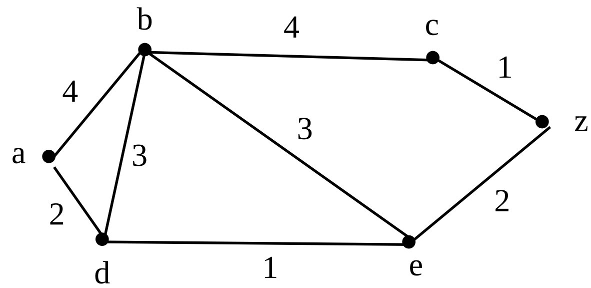


then we have a weighted graph. Come up with your own application of a weighted graph.

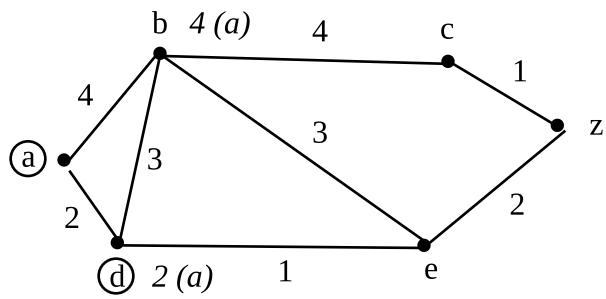
2. Once we have a weighted graph the problem is how to find the shortest path from one vertex to another. Suppose we have a simple complete graph - a graph with one edge between every pair of vertices. Then the *brute force* method of finding the shortest path between two vertices is to simply try every possibility. How many simple paths are there between two vertices in a complete graph with
   a. 3 vertices
   b. 4 vertices
   c. n vertices
   d. 10 vertices
   e. What is the order of finding the shortest path by brute force

3. From Problem (2) we see that an exhaustive search for the shortest path is $O(n!)$ and so gets very large very quickly. A more efficient algorithm is that developed by Dijkstra as illustrated in the following example -

   Suppose we want to find the shortest path in the following weighted graph from *a* to *z*

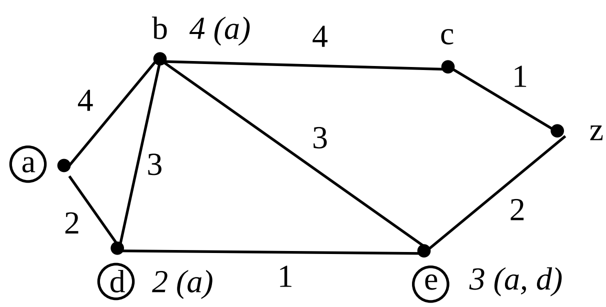


(1) Starting from vertex *a* we find and circle the closest adjacent vertex as follows



Note how we label the paths in the graph. And that we refer to the circled vertices as **distinguished vertices**

(2) We now take all vertices adjacent to distinguished vertices - and find the one with the shortest distance to *a*. Which in our graph is vertex *e* as follows



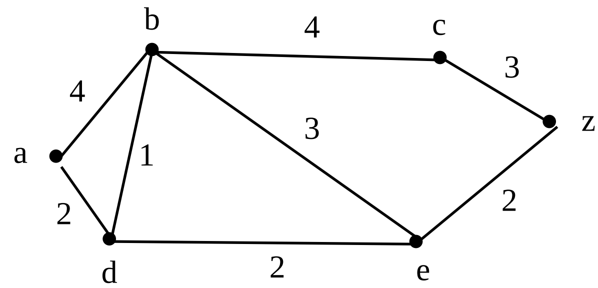
(3) We then continue until we reach vertex *z*

Complete Dijkstra's algorithm for this graph to find the shortest path from *a* to *z*. Show every step of the algorithm

4. Why do we just look at vertices adjacent to distinguished vertices to find the next distinguished vertex - the vertex with the next shortest path from the starting vertex

5. How is it possible to obtain more than one distinguished vertex at any given iteration of the algorithm

6. Use Dijkstra's algorithm to find the shortest path from vertex *a* to vertex *z* in the following weighted graph. Show every step of the algorithm



7. Use Dijkstra's algorithm to find the shortest path in a weighted graph that you make up. Show every step of the algorithm

8. How would you deal with a weighted graph that was not simple

9. What is the traveling salesman problem. How is it related to the shortest path problems of this Investigation