ECE 130 - GRAPHS - INVESTIGATION 16
GRAPHS OF RELATIONS

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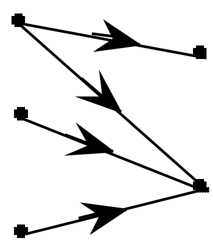
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last Investigation we introduced graphs G(V, E) with vertices V and edges E. The objective of this Investigation is to show how graphs can be used to give us graphical pictures of relations. Relations are of particular interest because they can be used to order information in data bases including search engines like Google.

1. We begin with a problem on functions. Draw a graph for a function from a set A to a set B that is
    a. Into but not onto
    b. Onto but not one-to-one
    c. Onto and one-to-one

2. From the last problem we know that if F is a function from a set A to a set B then for every element in A there is one and only one element in B. But more than one element in A can be mapped to the same element in B. **Relations R**, on the other hand, have graphs that look as follows



    a. Make use of this graph to explain how relations are different from functions
    b. Draw your own graph of a relation that is not a function

3. If R is a relation from a set A to a set B then for every element a in A that maps to an element b in B we have the ordered pair $(a,\ b)$ and write $a\text{R}b$. Draw the graph having the following relations: (0, b), (0, c), (1, a), (2, a), (2, b)

4. Suppose we write out all the ordered pairs of students and the classes they're taking (a separate ordered pair for each class)
    a. Under what circumstances are these ordered pairs a function
    b. Under what circumstances are these ordered pairs a relation

5. Suppose $R = \{(a,b) |\ a > b\}$ is a relation for the sets of integers A = {1, 2, 3, 4} and B = {2, 3, 4, 5}
    a. Find all the ordered pairs of R
    b. Draw the graph for R

6. A relation R is **reflexive** if $(a,a) \quad R$ for every element $a \quad A$. **Memorize** this definition and then find an example of a reflexive relation

7. Given a set S as follows S = {1, 2, 3}
    a. Write out ordered pairs for a reflexive relation among the elements of S
    b. Draw the graph for your relation in part (a)
    c. Write out the ordered pairs for a relation among the elements of S that is not reflexive
    d. Draw the graph for your relation in part (b)

8. A relation R is **symmetric** if whenever $(a,b) \quad R$ then $(b,a) \quad R$. **Memorize** this definition and then for S = {1, 2, 3}
    a. Write out the ordered pairs for a symmetric relation among the elements of S
    b. Draw the graph
    c. Write out the ordered pairs for a relation that is not symmetric
    d. Draw the graph

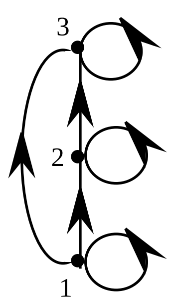
9. A relation R is **antisymmetric** if whenever both $(a,b) \quad R$ and $(b,a) \quad R$ then $a = b$
    a. Draw a graph of a relation that is antisymmetric
    b. Draw a graph of a relation that is not antisymmetric

10. A relation R is **transitive** if whenever $(a,b) \quad R$ and $(b,c) \quad R$ then $(a,c) \quad R$. **Memorize** this definition and then for S = {1, 2, 3}
    a. Write out the ordered pairs for a transitive relation among the elements of S
    b. Draw the graph
    c. How can you tell from a graph whether the relation is transitive
    d. Write out the ordered pairs for a relation that is not symmetric
    e. Draw the graph

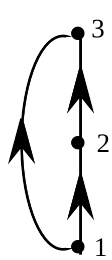
11. A relation R on a set S is called a **partial ordering** if it is reflexive, antisymmetric and transitive. Show that the relation $\leq$ is a partial ordering for S equal to the set of integers.

12. What is partial about a partial ordering

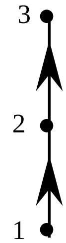
13. The objective of this problem is to illustrate how we can simplify the graphs of partial orderings. If we have a graph for a partial ordering as follows



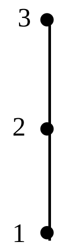
then we can remove all the loops at the vertices since every vertex has one as follows



And then we can remove all edges that are present as the result of transitivity as follows

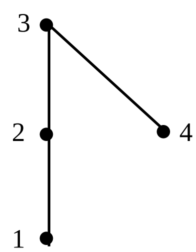


And then we can remove the arrows as follows as long as we put the vertices in the order the arrows are pointing as follows



We call the resulting graph a **Hasse diagram**. Given all this draw the Hasse diagram for the set S = {1, 2, 3, 4, 6} with the partial ordering $(a,b) \quad R$ if a divides b.

14. Can the following be a diagram



be a Hasse diagram. If so draw the corresponding graph of the relation. And if not explain why not