ECE 130 - GRAPHS - INVESTIGATION 15
INTRODUCTION TO GRAPHS

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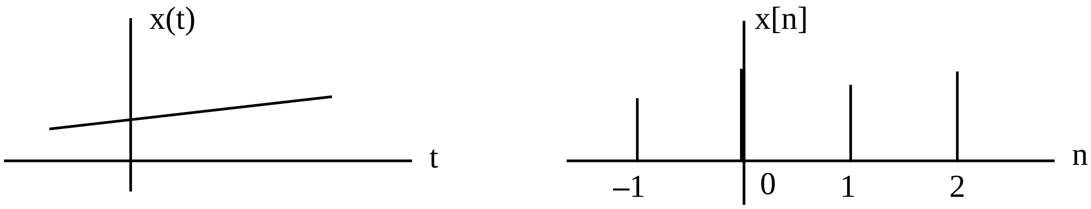
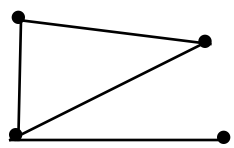
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

As we all know graphs of continuous and discrete functions like the following

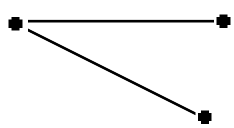


are great for showing relationships between variables. The objective of this and the next several Investigations is to introduce another kind of graph as follows



that is particularly useful for showing the relationships among both objects and discrete numbers

1. Graphs G(V, E) consist of **vertices V** and **edges E** connecting the vertices like the following example

   Note that the vertices are also referred to as *nodes* and the edges as *branches*. **Simple graphs** G(V, E) are graphs with

   (1) At most one edge connecting any two vertices
   (2) No loops from a vertex back to itself like the following

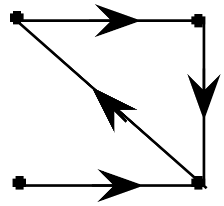
   

   a. Draw a simple graph with 4 nodes and 5 edges
   b. What's the most number of edges a simple graph with 4 nodes can have
   c. What's the most number of edges a simple graph with n nodes can have. Explain how you got your result

2. **Multigraphs** are graphs that can have more than one edge between any given pair of vertices. Draw a multigraph with 3 vertices and 5 edges

3. **Directed graphs** (also referred to as digraphs) are graphs with arrows on the edges like the

following



Note that graphs without arrows are referred to as undirected graphs. Suppose we want to use a graph to represent downtown streets with edges for the streets and vertices for the intersections. What kind of graph would you use if the streets were

a. One-Way
b. Two-Way

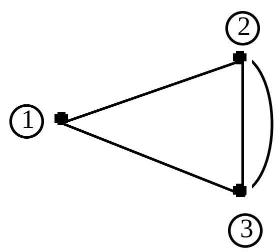
4. Give another example where you would use a directed rather than undirected graph

5. Draw a directed multigraph

6. How can a graph be used to show which students in a class are friends

7. Draw a directed graph for the results of a round robin tournament. What do the directions of your arrows mean

8. Given the following graph



with vertices having **degrees** (deg) as follows

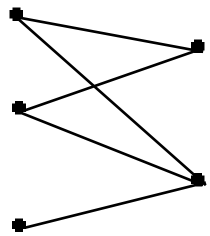
deg (1) = 2 deg (2) = 3 deg (3) = 3

a. What is the degree of a vertex
b. Draw a graph with 4 vertices having degrees 2, 3, 4, 5

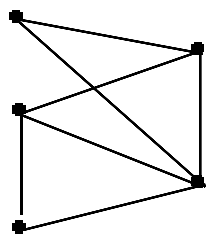
9. Given a graph with e edges and vertices v

a. Do an example to illustrate the fact that $2e = \sum_{v \in V} \deg(v)$. Note that this is referred to as the *handshaking theorem*
b. Explain in words why the above sum of the degrees is equal to $2e$
c. Why must the number of vertices with an odd degree be even

10. The following is an example of a **bipartite** graph



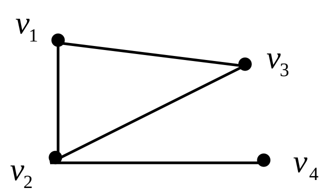
and here is an example of a graph that is not bipartite



a. What makes a graph bipartite
b. Draw a graph that is bipartite
c. Draw a graph that is not bipartite

11. A particularly useful way to specify simple graphs is with **adjacency matrices A** like the following

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$ for the graph



where the entries in the matrix corresponds to the vertices as follows

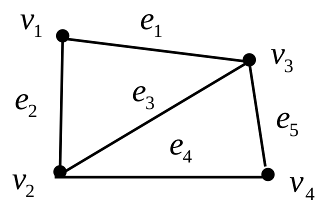
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

a. What does a 1 in row i and column j tell us. Note that we refer to this location as (i, j)
b. What does a 0 in (i, j) tell us

12. Now suppose graph G is a multigraph
a. How would you write the adjacency matrix
b. Do an example to illustrate

13. An alternate way to represent a graph is with **incidence matrices** like the following

$$I = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$ for the graph



where the rows correspond to the vertices and columns to the edges

a. What does a 1 in (i, j) tell us
b. What does a 0 in (i, j) tell us
c. Do an example to illustrate how to write an incidence matrix