ECE 130 - COUNTING - INVESTIGATION 14
PERMUTATIONS AND COMBINATIONS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In the last two Investigations we used a number of results including the counting rule and inclusion-exclusion to count various quantities. The objective of this Investigation is to count permutations and combinations.

1. Given a set S of three elements as follows $S = \{a, b, c\}$
    a. Complete the following list of the **permutations** of S

    $$a, b, c \quad a, c, b \quad b, a, c \quad . . .$$

    b. Verify that the total number of permutations of S is P(3) = 3!
    c. Explain in words why the total number of permutations of S is 3!

2. Explain in words what we mean by the permutations of a set S with n distinct elements

3. Generalizing on the result in Problem (1) we have that if a set S has n distinct elements then it has P(n) = n! permutations. **Memorize** this result. Then do an example to illustrate it

4. Again suppose that S is the set $S = \{a, b, c\}$
    a. Complete the following list of permutations of size 2

    $$a,b \quad b,a \quad b,c \ \cdots$$

    b. Verify that the total number of permutations of S of size 2 is $P(3,2) = 3 \ \ 2$
    c. Explain in words why $P(3,2) = 3 \ \ 2$

5. Generalizing on Problem (4) we have that if S has n distinct elements then it has

    $$P(n,r) = n(n-1)\cdots(n-r+1)$$

    permutations of size r. Do an example to illustrate this result with n=4 and r=2

6. Make use of the result in Problem (5) to show that $P(n,r) = \dfrac{n!}{(n-r)!}$

7. Suppose you want to visit five cities during Spring Break. How many different ways can you plan your trip

8. Suppose a league has 10 teams. What are all the ways we can have a 1st place team, 2nd place team and 3rd place team

9. When we calculate permutations as we've been doing we're counting **ordered** lists. For example a,b and b,a count as two permutations even though they both have exactly the same elements. **Combinations** on the other hand count just the number of lists with different elements. Order does not matter. For our set $S = \{a, b, c\}$
    a. Complete the following list of combinations $C(3,2)$ of size 2

$$a,b \quad a,c \ \cdots$$

b. Verify that the number of combinations of size 2 is equal to

$$C(3,2) = \frac{P(3,2)}{P(2,2)}$$

c. Explain in words why the expression in part (b) produces $C(3,2)$

10. Generalizing on Problem (9) we have that if S has n distinct elements then it has

$$C(n,r) = \frac{P(n,r)}{P(r,r)}$$

combinations of size r.

a. Do an example to illustrate this result with n=4 and r=2
b. Explain why the equation for $C(n,r)$ is true

11. Do an example to illustrate that $C(n,r) = C(n,n-r)$

12. Explain in words why $C(n,r) = C(n,n-r)$

13. Suppose we want to form a subcommittee of size 3 from a group of 10 professors. How many possibilities are there

14. Up to now all our calculations of permutations and combinations have been for sets with distinct elements. The objective of this problem is to see what happens when some of the elements are indistinguishable

a. Find all the binary numbers with three 1's and two 0's
b. Verify that your result in part (a) is equal to $\frac{5!}{3!2!}$

15. Generalize on the result of Problem (14) to explain why the number of permutations P of a set of n elements is as follows

$$P = \frac{P(n)}{P(n_1)P(n_2)} = \frac{n!}{n_1!\,n_2!}$$

when n_1 of the elements and n_2 of the elements are indistinguishable