ECE 130 - COUNTING - INVESTIGATION 13
SOME BASIC PRINCIPLES OF COUNTING

WINTER 2004 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

The main objectives of this Investigation are to review the product rule introduced in the last Investigation and then introduce the inclusion-exclusion principle and the pigeonhole principle.

1. A student must take one class from a list of 3 math classes and one class from a list of 4 physics classes. How many possible pairs of classes can the student take

2. Palindromes are strings like *101* and *dad* that stay the same when we reverse the order of the characters
   a. How many bit strings (strings of 1's and 0's) of length 3 are palindromes
   b. How many strings of letters of length 3 are palindromes
   c. How many bit strings of length 4 are palindromes
   d. How many bit strings of length n (n odd) are palindromes

3. The objective of this and the next problem is to review cardinality of a set as introduced in Investigation 2. Suppose we have two sets A = {a, b, c, d} and B = {b, d, e, f, g}. Find $|A \cup B|$ = cardinality of $A \cup B$ = equal to the number of elements in the union of A and B

4. Generalizing on the result of Problem (3) we have that

$$|A \cup B| = |A| + |B| - |A \cap B|$$

   This is referred to as the **inclusion-exclusion principle**. **Memorize** this result
   a. Explain in words why we subtract $|A \cap B|$
   b. Make up your own example to illustrate the inclusion-exclusion principle

5. Given 6 bit binary strings like the following

   110010

   a. How many begin with 1 and end with 0
   b. How many begin with 1 or end with 0

6. The **pigeonhole principle** is the intuitively obvious result that if we have more pigeons than pigeonholes then at least one pigeonhole has to have more than one pigeon. Now suppose we have n pigeonholes and m pigeons. How large does m have to be for us to be able to say that at least one pigeonhole has at least three pigeons.

7. Suppose we have a class of 30 students with

$$M_i = \text{Number of students with birthdays in month } i$$
$$M = \max\{M_i\}$$

   What's the smallest M can be