ECE 130 - COUNTING - INVESTIGATION 12
INTRODUCTION TO THE PRODUCT RULE

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

In a previous Investigation we showed that in order to calculate the complexity of sorting and searching algorithms - how long it takes to execute them - we need to count the number of comparisons. The objective of this and the next two Investigations is to develop methods for counting needed in applications ranging from the calculation of time complexity to probability to coding theory.

1. Suppose we label chairs with a letter and a number like A5, B17 and so on
   a. How many chairs can be labelled this way if the numbers N are in the range $1 \le N \le 10$
   b. How large does N have to be if there are 10^4 chairs

2. How many possible California license plates are there

3. Generalizing on the first two problems we have the **product rule** as follows

   If there are N_1 ways to do the first part of a "task" and N_2 ways to do the second part then there are $N_1 \ N_2$ ways to do the whole "task"

   Come up with a problem of your own to illustrate the counting principle.

4. Given a set A = {A1, A2} and a set B = {B1, B2, B3}
   a. How many functions are there from set A to set B
   b. Verify your result in part (a) by listing the functions

5. Make use of the product rule to generalize your result in Problem (4) to find the number of functions from a set A with m elements to a set B with n elements

6. Given a set A = {A1, A2} and a set B = {B1, B2, B3}
   a. How many one-to-one functions are there from A to B
   b. Verify your result in part (a) by listing the functions

7. How many one-to-one functions are there from a set A with n elements to a set B with n elements.

8. Given a set S with N elements
   a. How many subsets does S have (including the empty set). Hint: there are two possibilities for every element - it's either in the subset or not in the subset
   b. Verify your result in part (a) for the set S = {a, b}

9. Suppose the passwords for a given computer are 6 to 8 characters long. Find the total number of passwords
   a. If each character can be a lower case letter or a digit
   b. If each password must have at least one digit
   c. If each password must start with a letter and have at least one digit