ECE 130 - THE VERY BASICS - INVESTIGATION 1
INTRODUCTION TO SETS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include appropriate graphs and tables.

The overall goal of this inbtroductory class on discrete structures is to develop and evaluate algorithms for problems like putting numbers in numerical order, finding paths through networks and finding information in databases. We shall be particularly interested in finding data structures - ways of organizing and representing data - that make our algorithms easier to discover and implement. We begin with a review of sets.

1. As we all know we can specify a finite set S by listing the objects that belong to it as follows

   $$S = \{1,3,5\}$$

   Make a list of the elements in each of the following sets
   a. The vowels
   b. The colors of a traffic light
   c. The even integers between 1 and 9

2. More formally we define sets as follows

   A set is an unordered collection of objects.

   As it turns out this innocent looking definition - based on our intuitive notion of objects - can actually lead to contradictions as shown by Bertrand Russell. As a result we refer to set theory based on this definition as **naive set theory** in contrast to more rigorous axiomatic set theory. Be this as it may, naive set theory works fine for our everyday engineering applications. And so that is what we'll be using. Make use of the definition above to determine if the following two sets are equal

   $$A = \{1,2,3\} \qquad B = \{3,2,1\}$$

   How can you tell

3. If x is an object in the set S we write $x \quad S$. What are the objects in the following set S

   $$S = \{A, B, \{A, B\}\}$$

4. What is the empty set

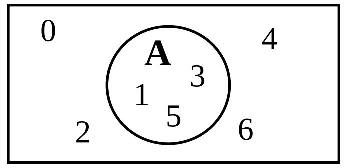
5. Up to now we've been specifying the elements in our sets S by listing the objects in them. Alternatively we can specify sets with statements like the following

   $$A = \{x \mid x \text{ is an integer and } -2 \le x < 3\}$$

   a. Write out in words what this statement for set A is saying
   b. Write out the list of objects in set A
   c. Come up with your own statement for a set B

6. Up to now we've been specifying sets with lists and descriptions. Alternatively we can draw

Venn Diagrams to obtain pictures of sets like the following



with $A = \{1,3,5\}$

a. What is meant by the **Universal Set**
b. What is the Universal Set for the Venn Diagram above
c. Draw a Venn Diagram for the set of even integers for a universal set consisting of all the integers from 1 to 9

7. The objective of this problem is to introduce subsets.

The set A is a ***subset*** *of B if and only if every element of A is also an element of B*

If A is a subset of B we write $A \quad B$

a. Write out in words the following symbolic definition of subset $\quad x(x \quad A \rightarrow x \quad B)$. Note that

means "for all" and $x \rightarrow y$ means that if x is true then y is also true

b. Come up with a subset A of the set $B = \{1,3,5,6,8\}$
c. Draw a Venn Diagram to illustrate your result in part (b)

8. What is a proper subset. What notation is used for a proper subset

9. A **power set** of a set A is the set of all subsets of A including the empty set. Write out the power set of the set $A = \{1,3,5\}$

10. The **cartesian product** of a set A and a set B denoted by $A \times B$ is the set of all ordered pairs (a,b) where $a \quad A$ and $b \quad B$. **Memorize** this definition. Then

a. Write out in words the following symbolic definition of a cartesian product

$$A \times B = \left\{(a,b) \middle| a \quad A \quad b \quad B\right\}$$

where means *and*

b. Write out the cartesian product of $A = \{a, b\}$ and $B = \{1,2\}$

11. How are cartestian products useful in the construction of databases

12. Do an example to illustrate that $A \times B \neq B \times A$ unless $A = B$

13. We define the **cardinality |A|** of a set A - the "size" of A - to be the number of elements in A

a. What is the cardinality of the set $A = \{1,3,5,6,8\}$
b. Find a set with infinite cardinality

14. As we saw in the last problem some sets can have an infinite number of elements. What's really surprising the first time you hear about it is that some infinite sets are "bigger" than others. The "smallest" infinite sets are those that we call **countable** - can be put in a list and so "counted" like integers as follows

$0, 1, -1, 2, -2, \ . . .$

Surprising as it may seem rational numbers are also countable. In particular the rational numbers from 0 to 1 can be listed as follows

$0, 1, 1/2, 1/3, 2/3, 1/4, 3/4, \ . . .$

a. Find the next five rational numbers in the above list
b. What are the rational numbers in the list just before and just after 10/11
c. From part (b) it can be concluded that a set is countable if we can come up with a scheme for putting it in an order where we can always specify the element right before and the element right after any given element x. Given this why would you expect there to be more irrational numbers than rational ones. In other words why would you expect the irrational numbers to be **uncountable**