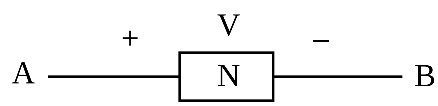
ECE 109 - KIRCHHOFF'S LAWS - INVESTIGATION 8
KIRCHHOFF'S VOLTAGE LAW

FALL 2006 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We know from our previous Investigations that the voltage drop V across a circuit element N as follows



is by definition

V ≡ (Potential at the node with the **plus**) - (Potential at the node with the **minus**)

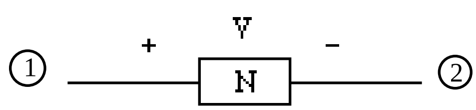
The voltage V tells us the difference between the potential energies of equivalent positive charge at the node with the plus and the node with the minus in the units of

volts = joules/coulomb.

We also know from previous Investigations that equivalent positive charges going from higher potentials to lower potentials are losing potential energy while those going from lower to higher potentials are gaining potential energy.

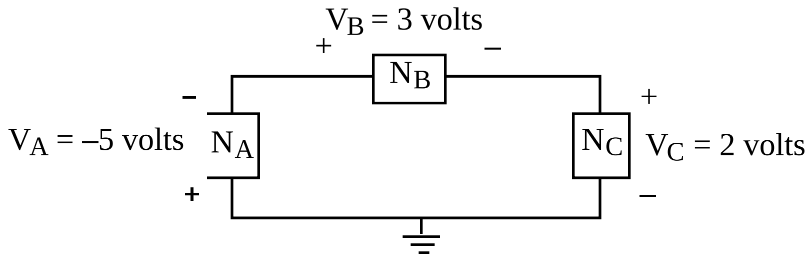
The objective of this Investigation is to come up with equations for the algebraic sums of voltages around closed loops analogous to Kirchhoff's Current Law at nodes. Be sure to take a look at the **Computer Demos** on Kirchhoff's Voltage Law.

1. We begin with a review problem. Given the following circuit element N with V = –5 volts



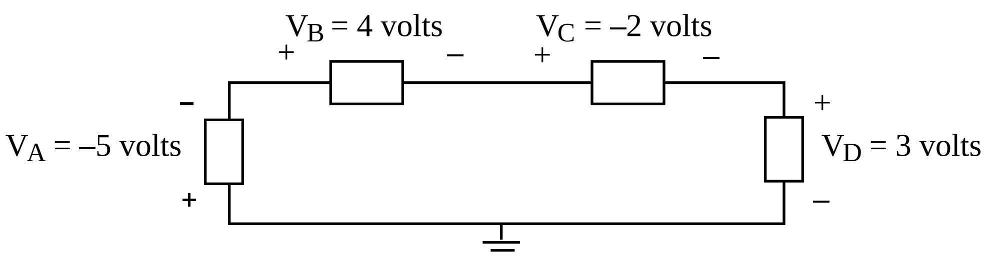
   a. Which node is at the higher potential
   b. Will epc flowing from node 1 to node 2 be receiving energy from N or transferring energy to it
   c. Will epc flowing from node 2 to node 1 be receiving energy from N or transferring energy to it

2. Now from the last Investigation we know that Kirchhoff's Current Law at nodes is based on the fact that charges always flow out of nodes at the same rate they flow in. The corresponding fundamental result for voltages is based on what happens to the energy the charges get from the sources as they flow through the circuit. As always we must start with results from the lab. Let's suppose, in particular, that we measure the following voltages



a. Make use of the measured voltages to calculate how much energy is transferred between each of the circuit elements N_A, N_B and N_C and Q = 2 coulombs of equivalent positive charge flowing clockwise around the circuit. Put your results in a Table with a column for whether the energy is being transferred from the epc to the circuit elements or from the circuit elements to the epc. Note that all your energies should be positive.
b. What is the net change in part (a) - if any - in the potential energy of the Q = 2 coulombs of equivalent positive charge after it flows completely around the circuit. Illustrate with a graph of the potential energy of Q as a function of its position as it travels clockwise around the circuit.
c. And finally, draw a graph of the potential (in volts) of equivalent positive charge as a function of position.

3. Generalizing on the result of Problem (2) it can be shown that equivalent positive charges flowing around a circuit always return with the same potential energy. The objective of this and the next several problems is to explore the consequences of this result. We begin with the special case of a circuit with all voltage reference directions **aligned** from plus to minus as we go clockwise around the circuit as follows



a. Draw a graph showing the potential of equivalent positive charge starting at the reference and flowing clockwise around the circuit.
b. Make use of your graph in part (a) to verify that equivalent positive charge going all the way around the circuit return with the same amount of potential energy/coulomb as when they started.
c. Find the sum of the positive voltages.
d. Find the sum of the negative voltages.
e. Now explain in words why

$$\sum \text{(Positive Voltages)} + \sum \text{(Negative Voltages)} = 0$$

for our circuit with *aligned* voltage reference directions. Hint - first explain why the two sums are equal in magnitude and then why they're opposite in sign.

4. From Problem (3) we know that if all the reference arrows are aligned as shown in the circuit diagram below then

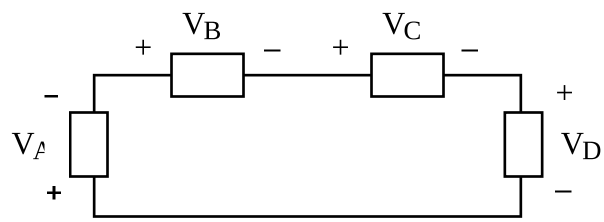
(1) Sum of all positive voltages = Number of joules/coulomb transferred by the epc to the resistors as they flow clockwise around the circuit

(2) Sum of all negative voltages = Minus number of joules/coulomb transferred to the epc by the sources as they flow clockwise around the circuit

Therefore, since the epc are transferring back to the circuit exactly the same amount of energy they're receiving we have that

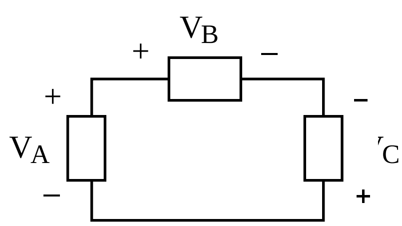
$$\sum_k V_k = 0 \quad \text{when all the voltages reference signs are aligned from plus to minus}$$

Now suppose equivalent positive charge flowing clockwise around the following circuit receive a total of 5 joules/coulomb. Assuming all the reference signs are *aligned* as follows

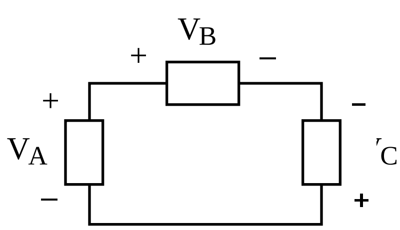


a. How many joules/coulomb did the equivalent positive charges return to the circuit.
b. What is the sum of the positive voltages.
c. What is the sum of the negative voltages.
d. What is the sum of all the voltages.

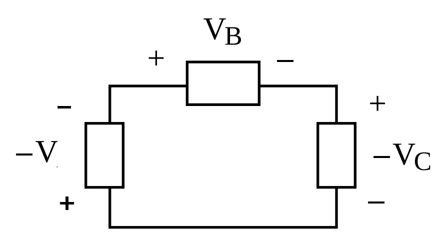
5. The objective of this problem is to find the relationship between the voltage drops in a circuit when they're not all aligned like in the following example



The trick is to make use of the fact that these non-aligned voltage reference directions can be turned around to form an equivalent set of aligned reference directions as follows



is equivalent to



Make use of this result to find the equation relating V_A, V_B and V_C

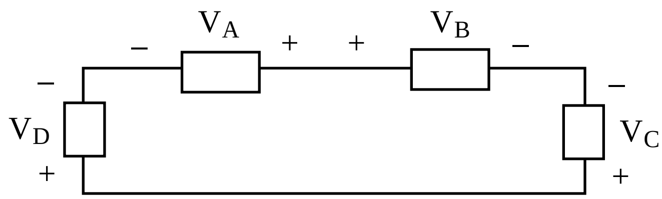
6. We refer to the equation from Problem (5) as follows

$$-V_A + V_B - V_C = 0$$

as the **algebraic sum of the voltages around the closed loop**.

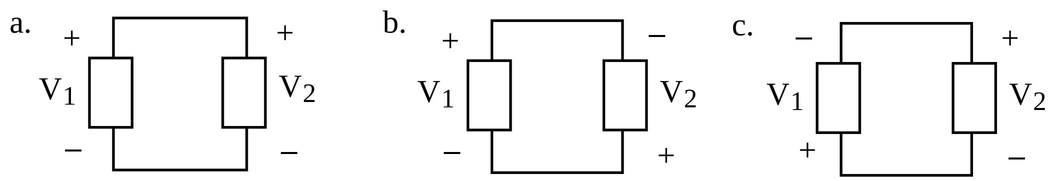
a. Which voltages do we add and which do we subtract when calculating the algebraic sum of the voltages as we go clockwise around a closed loop.

b. Now find the equation for the algebraic sum of the voltages around the following closed loop

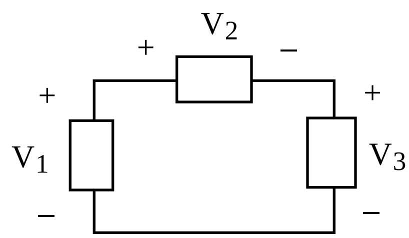


7. Generalizing on the results of Problems (5) and (6) we have that the algebraic sums of voltages around closed loops always add up to zero. We call this result **Kirchhoff's Voltage Law (KVL)**. **Memorize** it forever. Then make use of KVL to find V_B in Problem (6) if V_A = 3 volts, V_C = -2 volts and V_D = 2 volts

8. Find V_2 as a function of V_1 in each of the following circuits. **Memorize** your results



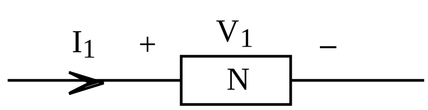
9. Given the following circuit



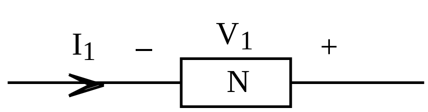
a. Find V_1 in terms of V_2 and V_3. **Memorize** this result

b. Find V_2 in terms of V_1 and V_3. **Memorize** this result

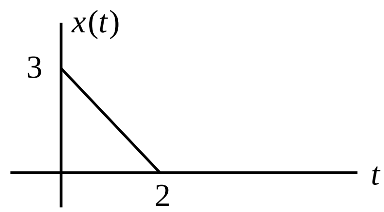
10. Suppose we measure V_1 = –3 volts and I_1 = –5 ma for the following circuit element



What would we have measured for V_1 and I_1 if their reference directions had been



11. Math Review - Given the following graph for $x(t)$



Sketch each of the following for $t \geq 0$

a. $y_1(t) = 2x(t)$
b. $y_2(t) = x(t) + 2$
c. $y_3(t) = 2x(t) + 2$