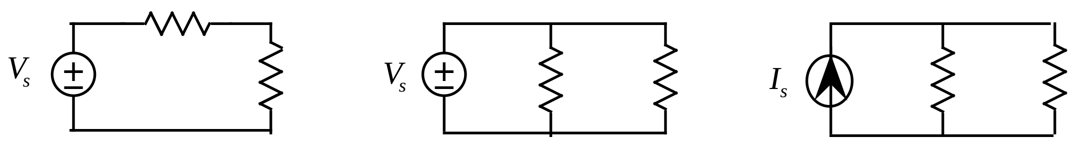
ECE 109 - KIRCHHOFF'S LAWS - INVESTIGATION 7
KIRCHHOFF'S CURRENT LAW

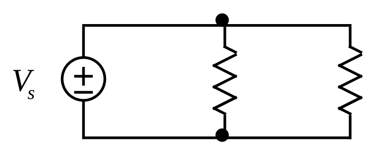
FALL 2006 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We've done a lot in the last couple of Investigations but all our circuits have had only one resistor and one source. To analyze more general circuits like the following

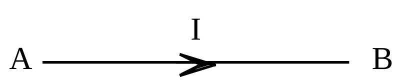


we need to know not only the circuit element values but also the constraints put on the voltages and currents by how the circuit elements are connected together. The objective of this Investigation is to introduce Kirchhoff's Current Law - a constraint on the values of the currents at the **nodes** - the points in our circuits where the wires come together as indicated by the dots in the following circuit



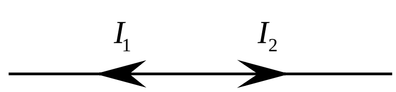
Be sure to take a look at the **Computer Demos** on Kirchhoff's Current Law.

1. We begin with two review problems. Given the following wire

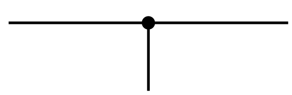


   a. How does reversing the direction of the reference arrow affect the sign of I.
   b. How does reversing the direction of the reference arrow affect the direction equivalent positive charge is flowing through the wire.

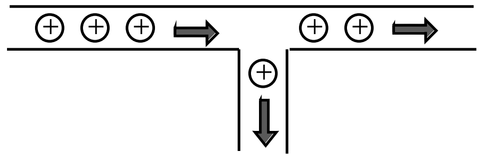
2. Explain why $I_1 + I_2 = 0$ in the following wire



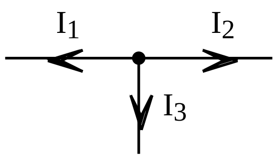
3. Now suppose we have the following node

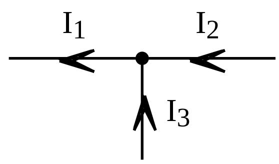


   with equivalent positive charge flowing as follows

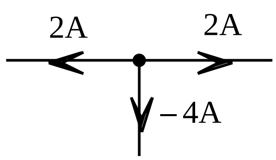


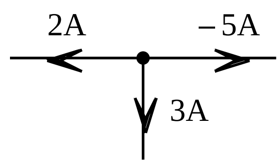
Assuming that each plus represents the flow of one coulomb/sec of equivalent positive charge, find the currents I_1, I_2 and I_3 for each of the following choices of the reference directions

a. 

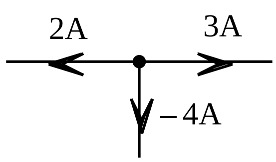
b. 

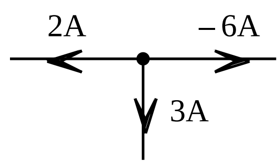
4. Now let us consider two nodes with currents that have been measured correctly as follows



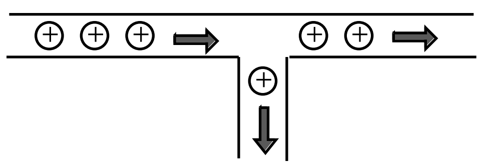


in contrast to two nodes where some of the currents have been measured incorrectly





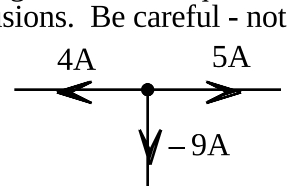
a. First draw enlarged pictures like the following

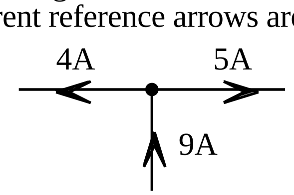


that show the flow of equivalent positive charge for **each** set of measurements above. Note that each plus corresponds to one coul/sec of current.

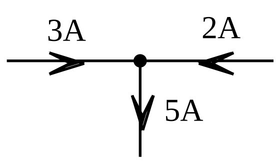
b. What you think the correct measurements imply is going on at nodes in contrast to what the incorrect measurements would have us believe.

5. Make use of your results in Problem (4) to determine which of the following nodes have currents that we know for sure have been measured incorrectly. Draw enlarged pictures showing the flow of equivalent positive charge like those in Problem (3) to support your conclusions. Be careful - not all the current reference arrows are pointing away from the nodes

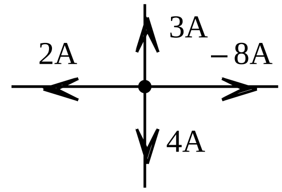
a. 

b. 

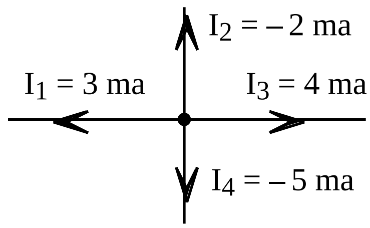
c.



d.



6. Generalizing on the results of Problems (4) and (5) it can be shown that equivalent positive charges always enter the nodes of our circuits at the same rate they're leaving. The objective of this and the next several problems is to explore the consequences of this result. We begin with the special case of all reference arrows **pointing away** from the node as follows



a. Draw a picture illustrating the flow of equivalent positive charge.
b. Make use of your drawing in part (a) to verify that equivalent positive charge is entering the node at the same rate (in coulombs/sec) it's leaving
c. What is the sum of the positive currents
d. What is the sum of the negative currents
e. Now explain in words why

$$\sum \text{(Positive Currents)} + \sum \text{(Negative Currents)} = 0$$

when all the reference arrows are *pointing away* from the node. Hint - first explain why the two sums are equal in magnitude and then why they're opposite in sign.

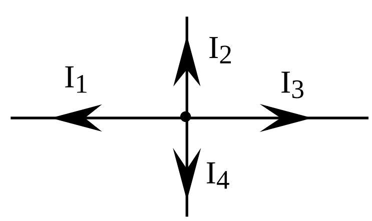
7. From Problem (6) we know that if all the reference arrows are pointing away from a node then

(1) Sum of all positive currents = Rate epc are leaving the node
(2) Sum of all negative currents = Minus the rate epc are entering the node

Therefore - since both rates are equal in magnitude - we have that

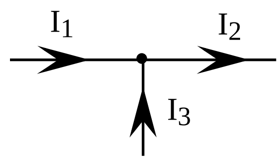
$$\sum_k I_k = 0 \quad \text{when all reference arrows are } \textit{pointing away} \text{ from a node}$$

Now suppose equivalent positive charge is entering the following node with all its reference arrows *pointing away* at the rate of $5x10^{-3}$ coulombs/sec

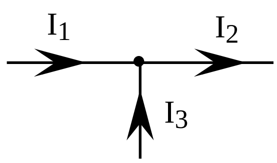


a. At what rate is which equivalent positive charge leaving the node
b. What is the sum of the positive currents
c. What is the sum of the negative currents
d. What is the sum of all the currents.

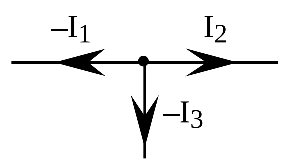
8. Generalizing on the results of Problems (6) and (7) we can show that when all the current reference arrows are *pointing away* from a node then the sum of all the currents is zero. **Memorize** this result. The objective of this problem is to see what happens when not all the reference arrows are pointing away from a node like in the following example



The trick is to make use of the fact that the reference arrows pointing into the node can be turned around to form an equivalent set of reference arrows pointing away as follows



is equivalent to



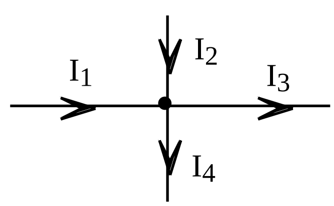
Make use of this equivalence to find the equation of I_1, I_2 and I_3 that adds up to zero.

9. We refer to the equation from Problem (7) as follows

$$-I_1 + I_2 - I_3 = 0$$

as the **algebraic sum of the currents leaving the node**.

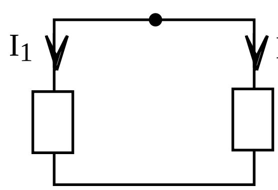
   a. Which currents do we add and which do we subtract when calculating the algebraic sum of the currents *leaving* a node.
   b. Find the equation for the algebraic sum of the current *leaving* the following node



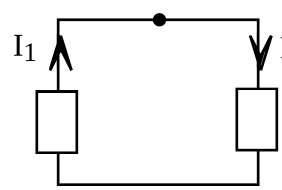
10. Generalizing on the results of Problems (8) and (9) we have that the *algebraic sums* of currents at nodes always add up to zero. We call this result **Kirchhoff's Current Law (KCL)**. **Memorize** Kirchhoff's Current Law forever. Then make use of it to find I_2 in Problem (8b) if I_1 = 3 ma, I_3 = -2 ma and I_4 = 2 ma

11. Make use of KCL to find I_2 as a function of I_1 in each of the following circuits. **Memorize** your results

a.

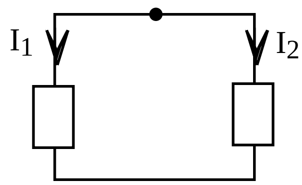


b.

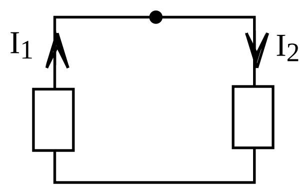


12. Given the following circuits

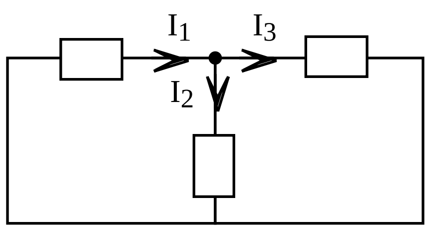
a. Find I_2 if $I_1 = 5$ ma



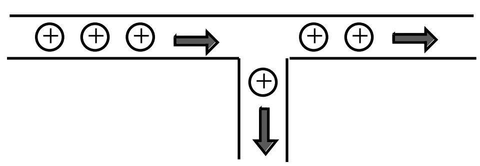
b. Find I_1 if $I_2 = 3$ma



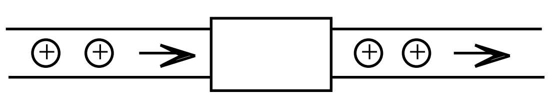
c. Find I_3 if $I_1 = 2$ma and $I_2 = 5$ma



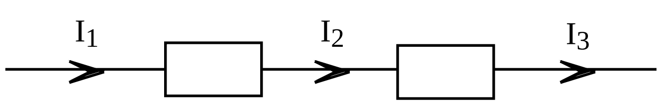
13. And now last, but not least, we need one more result. In the previous problems we've observed and made use of the fact that the rate equivalent positive charge is flowing into any given node like the following



is the same as the rate it's flowing out. This also turns out to be true for circuit elements. No matter what's in the following box - a battery, a light bulb or whatever equivalent positive charge will be flowing in at the same rate it's flowing out as follows

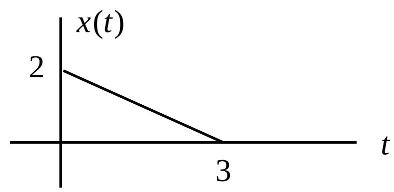


Make use of this fact to find the relationship between I_1, I_2 and I_3 in the following circuit



Justify your result. Draw a picture like that above to illustrate what's going on

14. Math Review - Given the following graph for $x(t)$



Sketch each of the following for $t \geq 0$

a. $y_1(t) = 2x(t)$
b. $y_2(t) = x(t) + 2$
c. $y_3(t) = 2x(t) + 2$