ECE 109 - TRANSFER FUNCTIONS - INVESTIGATION 25
SUPERPOSITION IN RESISTOR CIRCUITS

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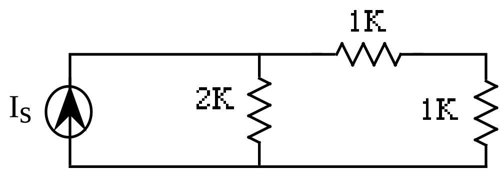
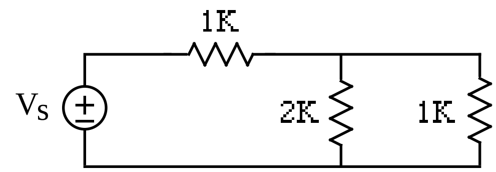
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

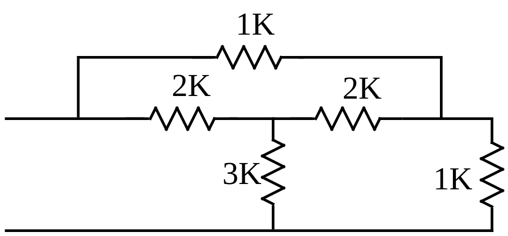
From our Investigations on transfer functions we know that every voltage and every current in resistor circuits with one source like the following



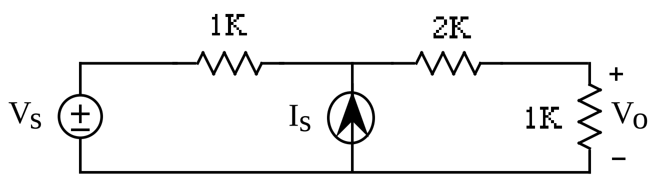


is proportional to the source. The objective of this Investigation is to see what happens when our circuits have more than one source. Be sure to take a look at the **Computer Demos** on Superposition.

1. We begin with a review problem. Find R_{EQ} of the following circuit



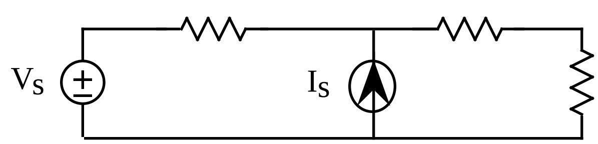
2. Now suppose we have a circuit with two sources as follows



Use node equations to find V_o in terms of the two sources V_s and I_s. Your result should be a sum of two terms - one proportional to V_s and the other to I_s

3. Repeat the analysis of Problem (2) for a circuit that you make up that contains two sources.

4. Generalizing on the results of Problems (2) and (3) it can be shown that every voltage and current in a circuit with two or more sources can be expressed as a sum of terms - each one proportional to one of the sources. V_o in the following circuit, for example



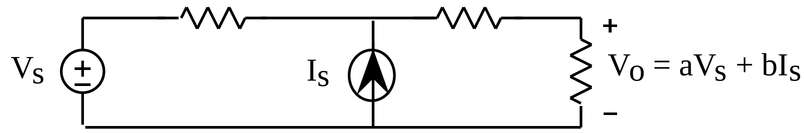
is equal to the sum of terms as follows

$$V_o = aV_s + bI_s$$

The objective of this problem is to find the physical significance of the terms aV_s and bI_s that add up to V_o

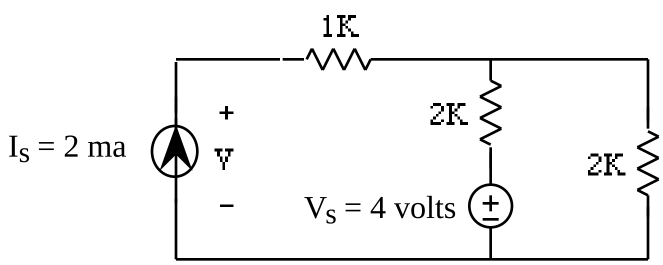
a. Make use of the equation above to find V_o as a function of V_s when $I_s = 0$
b. Draw the circuit for finding V_o when $I_s = 0$
c. Make use of the equation above to find V_O as a function of I_s when $V_s = 0$
d. Draw the circuit for finding V_o when $V_s = 0$
e. Make use of your results in parts (a)-(d) to explain how V_o can be found by first finding its response to V_s and then its response to I_s

5. From Problem (4) we know that we can find the response of a circuit with two sources like the following

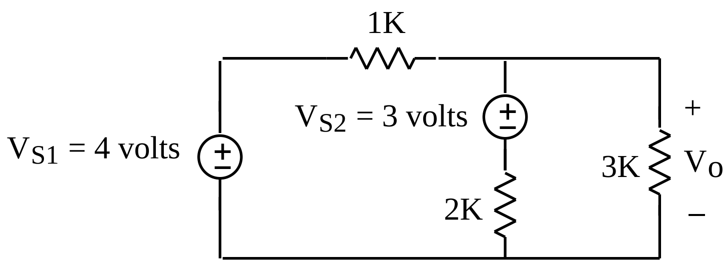


by first finding the response to V_s when I_s is set to zero and then finding the response to I_s when V_s is set to zero. We refer to this procedure as **superposition**. **Memorize** this result - its important because it says that each source contributes its own independent term to the output. Describe in words how superposition can be used to find voltages and currents in a circuit with n sources.

6. Use superposition to find V in the following circuit. Keep the bookkeeping straight by letting V_{01} equal the response to the current source and V_{02} equal the response to the voltage source. Be sure to **draw** the corresponding circuits



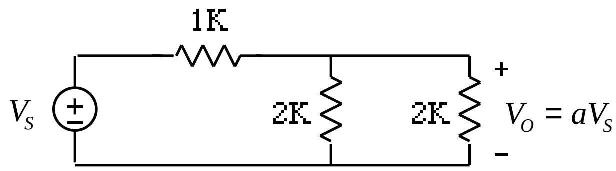
7. Use superposition to find V_O in the following circuit. Be sure to **draw** the corresponding circuits



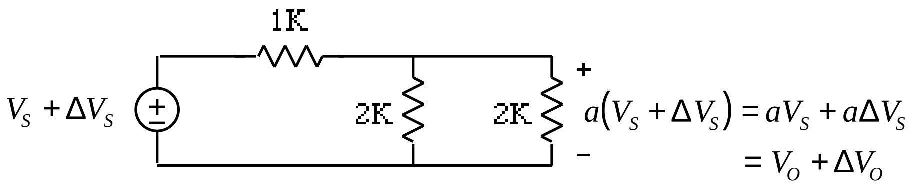
Hint - you can reverse V_{S2} and the 2K resistor without affecting any voltages or currents in the circuit

8. Find V_O in Problem (7) without using superposition. Which method was easier

9. As we saw in Problem (8), it's typically **easier** to analyze resistor circuits directly with node equations than to use superposition. But there are applications like the following where superposition is useful. Draw the circuit for finding the change ΔV_O in the output of the following circuit

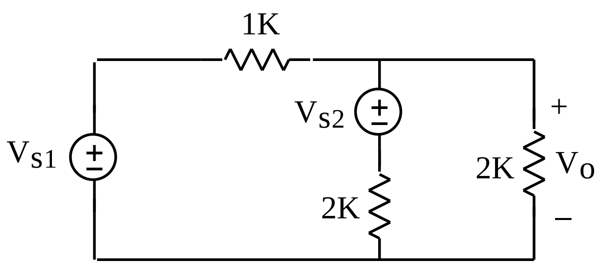


when V_S changes to $V_S + \Delta V_S$ as follows



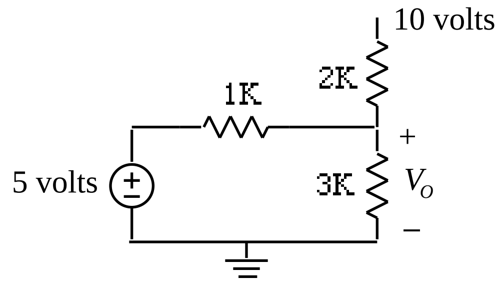
Hint - first redraw the circuit with the single voltage source $V_S + \Delta V_S$ replaced by a voltage source of value V_S in series with a second voltage source of value ΔV_S. And then make use of superposition to obtain the circuit for finding the response to ΔV_S

10. From Problem (9) we know that as a result of superposition we can find the change in the output of a linear resistor circuit due to a change in its input by simply calculating the response to the change. Now apply superposition to find the change in the output V_O of the following circuit

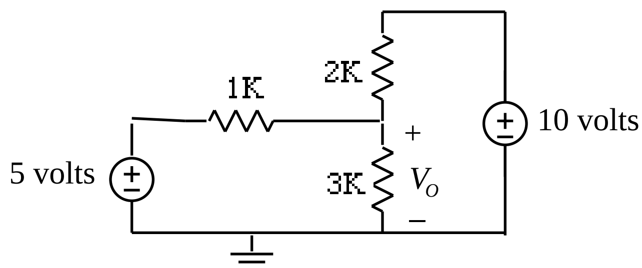


if V_{S2} increases by $\Delta V_{S2} = 0.5$ volts.

11. Use any method you wish to find V_O in the following circuit where



is shorthand for



12. Math Review - Sketch $x_1(t) = 2\cos(2\pi 100t + \pi/4)$ and $x_2(t) = 2\cos(2\pi 100t - \pi/4)$ on the same graph. Identify which is which